Combat System Maintenance Status

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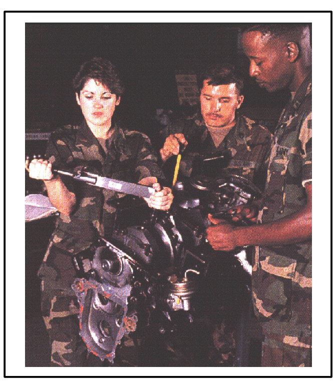
Introduction and setting

Army forces depend upon combat systems to move and fight. Because Army forces move in very stressful environments, these combat systems eventually break and require repair. While a system is awaiting repair, it is considered non-mission capable or NMC. The percentage of vehicles awaiting repair is one of the key indicators of a unit's ability to fight, and is tracked at all levels of command. The operational readiness rate, or OR Rate, is the percentage of systems in a given class which are Mission Capable (MC).



In this paper, we will discuss three simple models which describe the OR rate for the number of tanks in a tank battalion in the 1st Armored Division. We will also discuss ways the models could be extended to improve their usefulness to the commander.

We assume that the reader is familiar with elementary matrix algebra, linear systems of difference equations, and optimization using differential calculus.

Markov Chain models

We begin with a simple model where a tank can be in one of two states: mission capable (MC) or non-mission capable (NMC). Each day, on average a certain percentage of the mission capable tanks break, and move to the other state. Each day, a certain percentage of the NMC tanks are repaired and become operational. Graphically, we have:

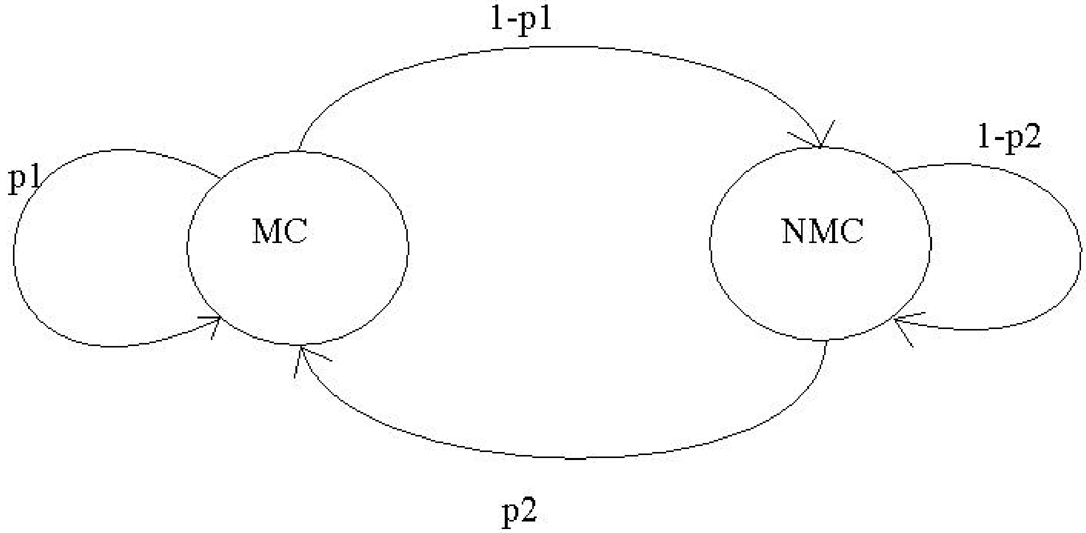


Figure 1

We use the notation that p_1 is the probability that a tank stays MC ; 1-p_1 is the probability that a tank moves from MC to NMC; p_2 is the probability that a tank moves from NMC to MC, and 1- p_2 is the probability that a tank stays NMC.

Let M_i be the number of tanks in the battalion which are mission capable on day *i*, and N_i be the number of tanks non-mission capable. Then we can write the expected transitions from one day to the next:

$$M_{i+1} = p_1 M_i + p_2 N_i$$
$$N_{i+1} = (1-p_1)M_i + (1-p_2)N_i$$

This system of equations can be easily expressed in matrix notation:

$$\begin{pmatrix} M_{i+1} \\ N_{i+1} \end{pmatrix} = \begin{pmatrix} p_1 & p_2 \\ 1-p_1 & 1-p_2 \end{pmatrix} \begin{pmatrix} M_i \\ N_i \end{pmatrix} = \begin{pmatrix} p_1 & p_2 \\ 1-p_1 & 1-p_2 \end{pmatrix}^{i+1} \begin{pmatrix} M_0 \\ N_0 \end{pmatrix}$$

This assumes that the p_i are known and constant, and ignores the randomness in this problem by just looking at the expected values of tanks operational. (Recall the expected value of an integer-valued variable is not necessarily an integer.)

This system will eventually reach a steady state, where the expected values do not change from day to day. The steady state can be found by eigenvector analysis. A matrix of probabilities such as we have constructed is called a probability transition matrix, since every entry is non-negative and every column sums to 1. It is known that every probability transition matrix has at least one eigenvalue equal to 1, and its associated eigenvector is the steady state for the system.

We can solve for the steady state eigenvector by solving the equation

$$\begin{pmatrix} M \\ N \end{pmatrix} = \begin{pmatrix} p_1 & p_2 \\ 1-p_1 & 1-p_2 \end{pmatrix} \begin{pmatrix} M \\ N \end{pmatrix}$$

since at the steady state the number of vehicles breaking exactly balances the number of vehicles being repaired.

We obtain the eigenvector

$$\begin{pmatrix} \dfrac{T\,p_2}{p_2 - p_1 + 1} \\ \dfrac{T\,(1-p_1)}{p_2 - p_1 + 1} \end{pmatrix},$$

where T is the total number of tanks in the battalion $(T = M_i + N_i)$. Regardless of our starting state, the top component of the eigenvector tells us how many vehicles, on average, we can expect eventually to have working, the bottom component tells us how many we expect to be awaiting repair.

The MC/NMC rate steady states can be found similarly to be

$$\begin{pmatrix} \dfrac{p_2}{p_2 - p_1 + 1} \\ \dfrac{(1-p_1)}{p_2 - p_1 + 1} \end{pmatrix}.$$

For example, let's let $p_1 = .95$ and $p_2 = .8$. This means that only 5% of the working vehicles break each day, and 80% of the broken vehicles are repaired. We set $T = 58$. Substitution gives us a steady state of $\begin{pmatrix} 54.5882 \\ 3.4118 \end{pmatrix}$, and an eventual steady state OR of 94.1176%. Of course, we can't have 54.5882 tanks. That is the expected value for the number operational, which is an average. Just as the average of 1 and 2 is 1.5, which is not an integer, so can the average number of tanks operational also not be an integer.

We can verify these calculations by constructing a spreadsheet model of the tank maintenance status. Let's start with M = 50 and N = 4. Figure 2 is a picture of how the system on average would behave.

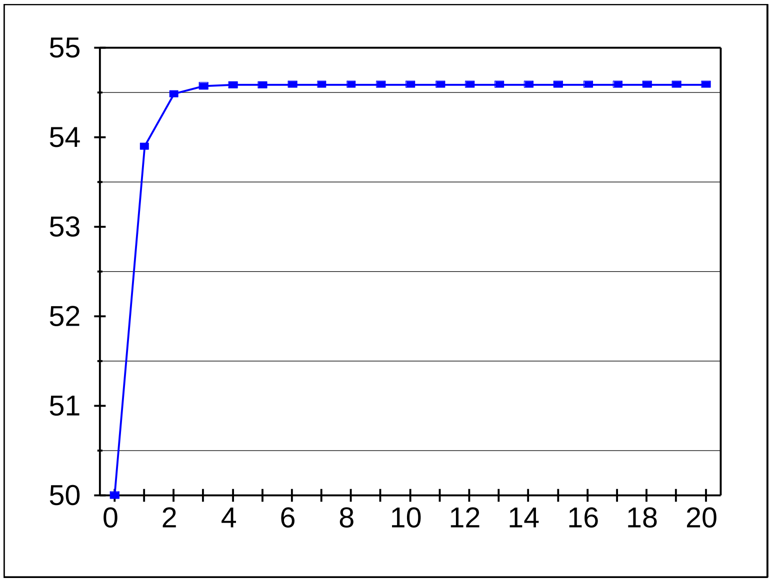


Figure 2

Let's look a little closer at the steady state for *M*, which we said was equal to

$$M_{steady\ state} = \frac{T\ p_2}{p_2 - p_1 + 1}.$$

If we want to improve the expected steady state for *M*, should we try to increase p_1, p_2, or some combination of the two? One way to gain insights into this question is to look at the gradient of steady state for *M* with respect to p_1 and p_2.

The gradient of M is given by:

$$\nabla M_{steady\ state} = \left(\frac{\partial M}{\partial p_1}, \frac{\partial M}{\partial p_2}\right) = \left(\frac{T\ p_2}{(p_2 - p_1 + 1)^2}, \frac{T(1 - p_1)}{(p_2 - p_1 + 1)^2}\right).$$

We know from calculus that the direction of greatest increase in M is in the direction of the gradient. That means to obtain the greatest increase in M for a fixed size change in the values of p_1 and p_2, the change in p_1 should be proportional to p_2, and the change in p_2 should be proportional to $1-p_1$. Does this make sense?

If we are repairing vehicles quickly already, then it makes sense to work on not breaking them instead of working on improving our repair time. That is why the change in p_1 is proportional to p_2. As p_2 increases, we work more on p_1.

If we are breaking vehicles frequently, it makes sense to improve our repair capability. $(1-p_1)$ is the fraction of vehicles expected to "break" each time period. As that increases, so should p_2, according to the gradient.

Notice that unless p_2 is zero, or p_1 is one, we should work on improving both percentages. The gradient tells us how to allocate our effort between them.

There are at least two shortcomings of this model. First, it is deterministic: we only work with the expected number of vehicles in each category, and this ignores the randomness of the true behavior, as well as the integer nature of the objects. We can't have 54.5882 tanks mission capable: we can only have 54 or 55. We don't know from this model how much variation to expect around the average.

The second shortcoming is that it is very simple. It doesn't recognize that vehicles break at different rates depending upon whether one is driving them in the field or not. Vehicles are repaired at different rates depending upon the availability of spare parts and mechanics, and upon how many other vehicles are waiting to be repaired. The availability of parts depends on the priority code of the unit. If the unit level maintenance can not repair the vehicle, it is sent to higher level maintenance, which has different repair rates. A better model would incorporate these additional features.

However, this simple model is useful. It allows us to gain understanding about our system, which is the hallmark of an effective model.

Binomial Equation

It is possible to improve the model in the previous section by considering a stochastic model. This will allow us to not only understand the average behavior of our model, but also how much the number of MC tanks varies around that average. There is a significant difference between 54.5882 (plus or minus .1), and 54.5882 (plus or minus 10). We will use the binomial distribution for our models.

Let's consider the number of tanks that are mission capable each day. That number will be the sum of those tanks that were mission capable the previous day and stayed in that state, plus the number of tanks that were repaired the previous day. This can be modeled as the sum of two binomially distributed random variables. Let M_i be the number of mission capable tanks the preceding period, and T be the total number of tanks. Then

$$M_{i+1} = Bin(p_1, M_i) + Bin(p_2, T - M_i) \, .$$

M_{i+1} is the sum of two binomial random variables, but it itself is not binomially distributed unless $p_1 = p_2$.

How does M_{i+1} behave? Let's imagine 20 battalions all starting at with the same number of MC and NMC tanks (54 and 4), each with $p_1 = 0.95$ and $p_2 = 0.80$, and each following our model. We can graph their number of MC tanks using a spreadsheet, and obtain Figure 3:

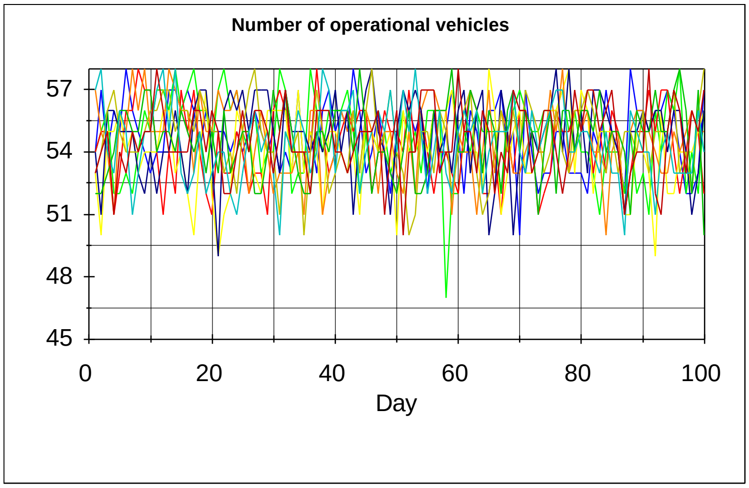


Figure 3

This image of 20 different histories (one for each of the battalions) gives us a feel for the variability we can expect in the number of operational tanks. We can expect to have between 51 and 58 tanks MC usually, and only about 1 in every 2000 days would we have as few as 47 tanks operational.

We can also use the spreadsheet to do some simple "what--if" analysis. We can put p_1 and p_2 in cells, and have all the calculations refer to those cells. Then as we vary the values of those cells, we can dynamically see the effect in operational readiness rate. At Figure 4, we see an example, with $p_1 = .85, p_2 = .5.$:

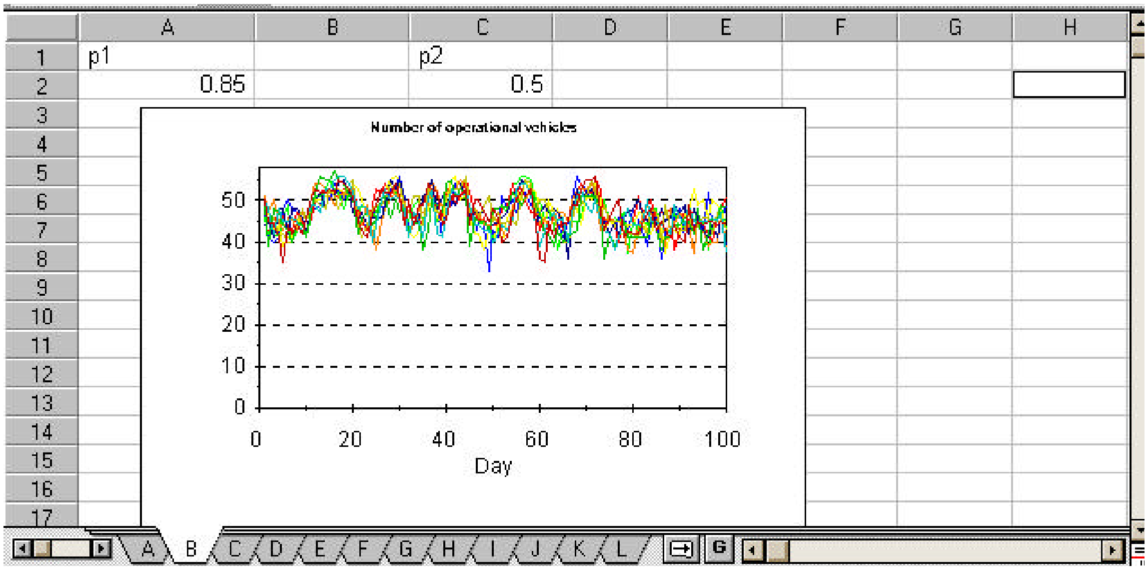


Figure 4

Notice that this data is serially correlated. If the previous day was low, the next day tends to be low, as well. If the previous day was high, the next day tends to be high. It is obvious from the graph that the day to day levels of M_i are not independent. Of course, we knew that since in our model the distribution of M_{i+1} was a function of M_i.

This type of pictorial representation of a stochastic discrete system is very useful to demonstrate the properties of a maintenance system to a commander. Pictures tell stories that mere numbers often fail to convey. The picture of the Markov chain model shows only the long-term trend of the average number of tanks. These pictures from the simulation show both the long-term average and the variability. While the average number of tanks MC may reach an equilibrium value, the variability of the actual number of tanks up at any given time does not go to zero! The randomness remains.

Estimating the parameters

How do we determine p_1 and p_2? We may never know them exactly. The best we can hope to do is to estimate them from historical data. We have assumed in our earlier models that p_1 and p_2 are constant. This is a very strong assumption. If it holds, we can estimate p_1 and p_2 from prior records. In the section after this, we discuss an approach for relaxing the assumption of constant transition probabilities.

For now, we continue to assume that the transition probabilities are constant. How may we estimate them?

Our answer will depend on what type of data we have available. If we have data that tells us for each day the number of vehicles that stayed MC, that went from MC to NMC, that went from NMC to MC, and stayed NMC, we can estimate the transition probabilities directly. Unfortunately, the data we have from the 1st Armored Division only has the number of MC vehicles for each reporting period.

We can use that data to estimate p_1 and p_2 in our binomial model by constructing a discrete dynamical system of expected values under our binomial model where we allow the transition probabilities to be variables. We have that

$$E\left(M_{i+1}\right) = p_1 M_i + p_2 N_i \,,$$

where E(M) means the expected value of M. We then calculate residuals, which are the difference between the expected value and the observed values. We square the residuals, and choose the two values of p_1 and p_2 that minimize the sum of the squared errors. Using the **Solve** macro from Excel, we find the values of p_1 and p_2 that minimize the squared residuals. The output is shown in Figure 5.

The spreadsheet is searching through all possible values of p_1 and p_2 to find the values that minimize the sum of squared residuals. These values are all listed on the first line of the output. We see that the best estimate from this data is that $p_1 = 96.66\%$ and $p_2 = 32.11\%$.

	p1	p2		sum
	0.966653	0.321071		113.5452
Day	M	N	EM	r2
36	55	3		
35	54	4	54.12913	0.016674
34	54	4	53.48355	0.266725
33	52	6	53.48355	2.200907
32	52	6	52.19238	0.03701
31	52	6	52.19238	0.03701
30	52	6	52.19238	0.03701
29	53	5	52.19238	0.652249
28	53	5	52.83796	0.026256
27	53	5	52.83796	0.026256
26	53	5	52.83796	0.026256
25	53	5	52.83796	0.026256
24	53	5	52.83796	0.026256
23	53	5	52.83796	0.026256
22	50	8	52.83796	8.054035
21	53	5	50.90122	4.404893
20	53	5	52.83796	0.026256
19	48	10	52.83796	23.40589
18	52	6	49.61005	5.711853
17	53	5	52.19238	0.652249
16	53	5	52.83796	0.026256
15	53	5	52.83796	0.026256
14	48	10	52.83796	23.40589
13	49	9	49.61005	0.372163
12	49	9	50.25563	1.576617
11	49	9	50.25563	1.576617
10	48	10	50.25563	5.087885
9	53	5	49.61005	11.49175
8	53	5	52.83796	0.026256
7	53	5	52.83796	0.026256
6	54	4	52.83796	1.350329
5	55	3	53.48355	2.299634
4	58	0	54.12913	14.98365
3	58	0	56.06587	3.74084
2	55	3	56.06587	1.136089
1	55	3	54.12913	0.758418

Figure 5

Modeling the transition probabilities as functions of other variables

Let's look at a plot of the residuals from the previous section, given at Figure 6:

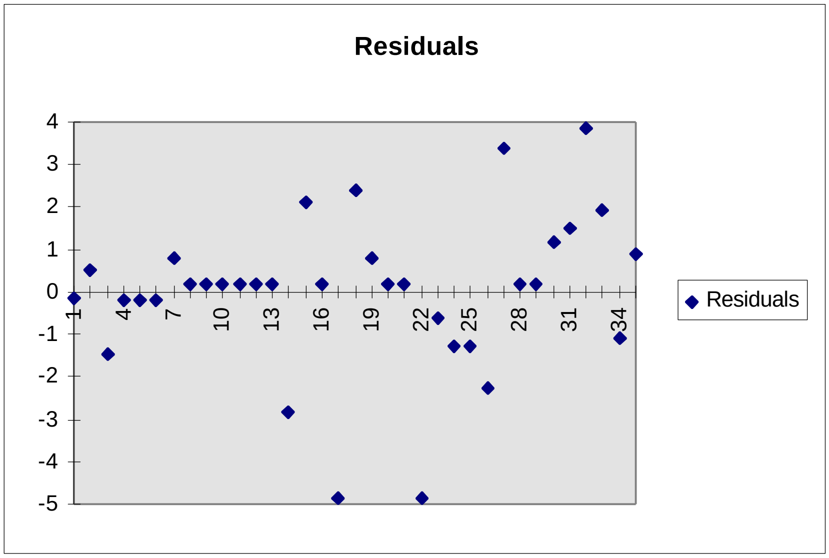


Figure 6

We see that there are some days where the residuals are much bigger than the others are, indicating that our estimated model does not fit too well on those days. Can we find information to explain why the model does poorly on those days?

One approach is to allow p_1 and p_2 to be functions of some other variables. For example, p_1 might very well be affected by whether or not the battalion tanks were driven that day. p_2 might be affected by the number of mechanics available, the number of spare parts available, and even the number of NMC vehicles: the more broken, the less likely any one of them might be fixed.

We were not able to collect data on these predictors, but we can find the days the battalion was in the field using its vehicles. To illustrate the method, we add a predictor to our data set with value 1 if the battalion drove the vehicles in the field that day, and value 0 if not. It is still possible to have a vehicle become NMC if it is not driven because of the preventive maintenance checks and services (PMCS), but it is much less likely.

We define p_3 as the amount that p_1 changes when the battalion is in the field. We then have $EM_{i+1} = (p_1 + p_3 I_i) M_i + p_2 N_i$, where I_i is an indicator variable equal to 1 if the battalion is in the field that day. Just as before, we use Excel to solve for all three p values, and obtain the spreadsheet output at Figure 7, below:

	p1	p2	p3	sum		
	0.950452	0.597506	-0.07669	57.00474		day in field?
Day	M	N	EM	r2	r	
36	55	3	53.71445	1.652645	1.285552	0
35	54	4	53.71445	0.08154	0.285552	0
34	54	4	53.00855	0.982964	0.991445	0
33	52	6	53.00855	1.017182	-1.00855	0
32	52	6	53.00855	1.017182	-1.00855	0
31	52	6	53.00855	1.017182	-1.00855	0
30	52	6	53.3615	1.853685	-1.3615	0
29	53	5	53.3615	0.130683	-0.3615	0
28	53	5	53.3615	0.130683	-0.3615	0
27	53	5	53.3615	0.130683	-0.3615	0
26	53	5	53.3615	0.130683	-0.3615	0
25	53	5	53.3615	0.130683	-0.3615	0
24	53	5	53.3615	0.130683	-0.3615	0
23	53	5	52.30266	0.486281	0.697339	0
22	50	8	53.3615	11.29969	-3.3615	0
21	53	5	53.3615	0.130683	-0.3615	0
20	53	5	51.59677	1.969059	1.403232	0
19	48	10	49.02076	1.041956	-1.02076	1
18	52	6	53.3615	1.853685	-1.3615	0
17	53	5	53.3615	0.130683	-0.3615	0
16	53	5	53.3615	0.130683	-0.3615	0
15	53	5	51.59677	1.969059	1.403232	0
14	48	10	48.19199	0.036859	-0.19199	1
13	49	9	48.19199	0.652884	0.808012	1
12	49	9	48.19199	0.652884	0.808012	1
11	49	9	47.91573	1.175642	1.08427	1
10	48	10	49.29702	1.682263	-1.29702	1
9	53	5	53.3615	0.130683	-0.3615	0
8	53	5	53.3615	0.130683	-0.3615	0
7	53	5	53.71445	0.510436	-0.71445	0
6	54	4	54.06739	0.004542	-0.06739	0
5	55	3	55.12623	0.015935	-0.12623	0
4	58	0	55.12623	8.258531	2.873766	0
3	58	0	54.06739	15.46539	3.932606	0
2	55	3	54.06739	0.869753	0.932606	0
1	55	3				0

Figure 7

Notice how much our estimates change with the addition of the new information, and how much our squared error is reduced. The plot of the residuals at Figure 8 indicates a much better fit, although there still are some days which are not well explained by the model.

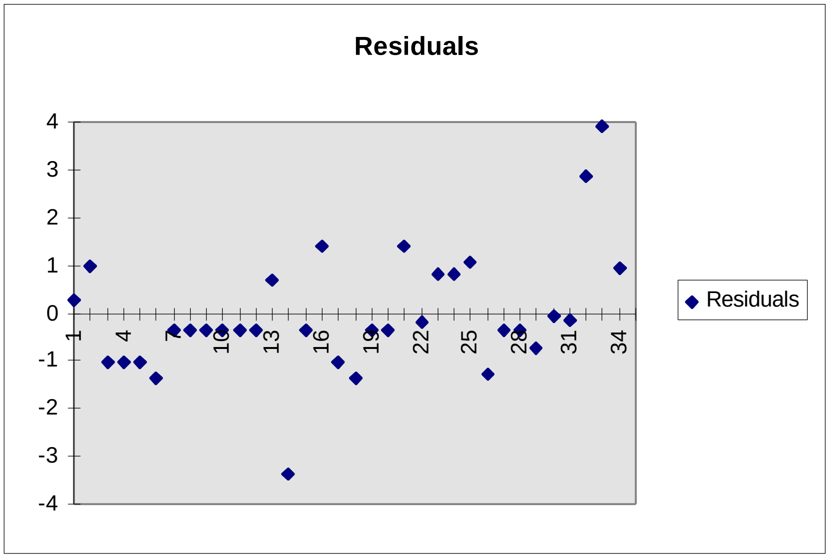


Figure 8

With more information, we might model p_1 and p_2 as functions of the other predictors, and fit them using least squares. We could then not only have better estimates of the parameters, but also an understanding of how different factors affect the readiness of the battalion's vehicles.

Conclusion

These simple models are very useful to leaders trying to understand the usual variation of their systems. We have been able to provide this battalion commander with much useful information about the OR of his unit. The estimate of p_3, in particular, helps the commander quantify the readiness cost of field training.

It is very satisfying to see in this problem that the discrete dynamical systems, calculus, and statistics topics of the USMA core curriculum are so directly useful to the Army in the field. We have seen that the expected number of operational vehicles can be modeled with a first order difference equation. We have used the binomial distribution from the probability and statistics course to understand the variability of the readiness rate. Minimizing the sum of squared errors to obtain estimates is a topic from both the calculus and statistics. Finally, our use of the gradient to see the relationship between changes in the parameters and changes in the OR rate is a straightforward application of the multivariable differential calculus.

Exercises

1. Your battalion values for p_1 and p_2 are given by .95 and .7, respectively. Assume you initially have 58 vehicles, 45 which are MC.

a) Formulate the transition matrix for MC and NMC tanks for your battalion.

b) Find the steady state readiness posture of your battalion. How long does it take to achieve this?

c) Use simulation to estimate the variability of the MC and NMC rates.

d) Do these values of p_1 and p_2 produce pronounced serial correlation? Why or why not, and why would a commander care?

2. You obtain the data in Figure 9 on your tank battalion's MC and NMC counts. Estimate p_1 and p_2. Interpret them in a short paragraph to your commander. Include a simulation that demonstrates how variable the MC and NMC daily totals can be.

Day	MC	NMC
0	40	18
1	43	15
2	44	14
3	46	12
4	50	8
5	51	7
6	50	8
7	53	5
8	55	3
9	56	2
10	52	6
11	52	6
12	50	8
13	53	5
14	54	4
15	56	2
16	54	4
17	53	5
18	52	6
19	52	6
20	49	9

Figure 9

3. You have the data at Figure 10 from a sister battalion, which also includes whether or not they were in the field. Estimate p_1, p_2, and p_3. Summarize your findings to your commander in a short report. Include appropriate graphical evidence.

In field?	MC	NMC
0	40	18
0	51	7
0	54	4
0	51	7
0	48	10
0	50	8
0	51	7
1	34	24
1	38	20
1	33	25
1	39	19
1	42	16
1	40	18
1	36	22
1	38	20
0	51	7
0	49	9
0	50	8
0	55	3
0	55	3
0	54	4

Figure 10

4. Your commander is uncertain about this model. Write a short essay explaining the assumptions of this Markov chain maintenance model. Based on your experiences, discuss which assumptions might be reasonable and which might be unreasonable. Cite examples to support your discussion. Give enough information so an informed decision can be made about using the model in your unit. Write in language your commander can understand.

References

1. Army Regulation 750-1, Preventive Maintenance Checks and Services

2. Field Manual 43-5, Unit Maintenance Operations

3. Department of the Army Pamphlet 738-750, Army Maintenance Management System

4. Department of the Army Pamphlet 750-35, Guide for Motor Pool Operations