



A new ordered family of Lorenz curves with an application to measuring income inequality and poverty in rural China

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ABSTRACT

The most common data source on income distribution in China is grouped data. When income data is in grouped form, some acceptable Lorenz model is needed to approximate the underlying Lorenz curve. This paper presents a new family of Lorenz curves and applies the main model in our proposed family of Lorenz curves to income data for rural China over the period 1980 to 2006. We find that the income share of the rural population at the low end of the income scale has been shrinking, income inequality in rural China has increased over time and that income inequality has impeded attempts to reduce poverty. However, the welfare of the rural population is still improving in terms of the generalized Lorenz dominance criterion.

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1. Introduction

China has undergone monumental economic transition since 1979, which has resulted in a high rate of economic growth. The distribution of income has also gone through large-scale change. There have been several studies of income inequality in rural China (see eg. Benjamin, Brandt, & Giles, 2005; Griffen & Zhao, 1993; Hu, Wang, & Kang, 1997; Rozelle, 1996; Wan & Zhou, 2005; Yao, 1997). These studies point to a general trend of an increase in income inequality since the late 1970s. Rising rural income inequality threatens China's ability to maintain sustainable growth and potentially impinges on political and social stability (Wan & Zhou, 2005, p.107). The latter has been of particular concern to the Chinese government with income distribution a central platform of constructing a harmonious society as first enunciated by the Hu-Wen administration during the 2005 National People's Congress. Income inequality in China also has implications that extend beyond its national boundaries. As noted by Chotikapanich, Rao, and Tang (2007, pp. 127–128): "As China accounts for about a quarter of the world's population, changes in income and income inequality in China have important implications [for] global income inequality ... This means that any advancement in the measurement of income inequality within China is not only important for understanding the economic development and well-being of people inside the 'Middle Kingdom', but also important in the global context".

Studies which have examined income inequality in China have used one of the two main sources of data on income distribution in China; namely, household survey data or grouped data. While rich in detail, the release of household survey data has been sporadic and restricted to a few provinces at a time. Thus, studies of income inequality in China that have used household survey data have been forced to focus on specific geographic locales and isolated years (see eg. Gustafsson & Li, 2002; Meng, 2004). In contrast to household surveys, grouped data is more readily available and has a wider coverage. However, since the income data is in grouped form, some acceptable Lorenz model is needed to approximate the underlying Lorenz curve. Several parametric Lorenz

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models to fit grouped data have been developed (see Basmann, Hayes, Slottje, & Johnson, 1990; Cheong, 2002; Chotikapanich, 1993; Gupta, 1984; Holm, 1993; Kakwani, 1980; Kakwani & Podder, 1973, 1976; Ogwang & Rao, 1996, 2000; Ortega, Martin, Fernandez, Ladoux, & Garcia, 1991; Rao & Tam, 1987; Rasche, Gaffney, Koo, & Obst, 1980; Rossi, 1985; Ryn & Slottje, 1996; Sarabia, 1997; Sarabia, Castillo, Pascual, & Sarabia, 2005; Sarabia, Castillo, & Slottje, 1999, 2001; Schader & Schmid, 1994). A limitation of some models that have been used to approximate the Lorenz curve is that they do not in fact satisfy the definition of the Lorenz curve (see Kakwani, 1980; Kakwani & Podder, 1976). Another shortcoming of some Lorenz models, as noted by Cheong (2002), is that they tend to underestimate or overestimate the Gini index by generating values outside the Gastwirth (1972) lower or upper bounds for the Gini index. Recent research has concentrated on finding models which are satisfactory both in theory and in practice. These models need to be able to satisfy the definition of the Lorenz curve as well as exhibit good performance for a variety of data sources (see, eg, Ogwang & Rao, 2000; Ryn & Slottje, 1996; Sarabia et al., 1999).

The objective of this paper is to develop a new Lorenz curve model, building on a basic model, first proposed by Sarabia et al. (1999), and apply this model to examine rural income inequality and poverty with grouped data in China. Sarabia et al. (1999) suggest a basic model of the form $\tilde{L}(p) = p^\alpha L(p)^\eta$. We first show that this model can be generalized and proceed to apply the generalized model to income data for rural China over the period 1980 to 2006. This allows us to examine the relationship between rural income inequality and poverty over a quarter century of market reforms. To this point: “One striking feature of the current discussion of inequality in China is the absence of well-documented facts about inequality and temporal changes in the structure of the income distribution” (Benjamin et al., 2005, p.769). This reflects the fact that while there are several studies of income inequality in China, with few exceptions most have provided only a snapshot without a time profile. Our results show that the income share of low income earners in rural China is getting smaller, income inequality in rural China has increased over time and that rural income inequality has impeded attempts to reduce poverty. However, in spite of worsening rural income inequality, we also find that the welfare of the rural population is still improving in terms of the generalized Lorenz dominance criterion.

The paper is set out as follows. Section 2 presents a new Lorenz curve model, building on a basic model, first proposed by Sarabia et al. (1999). Section 3 outlines the indices used to measure income inequality and poverty in rural China and generalized Lorenz dominance used to evaluate changes in social welfare over time. Section 4 applies the new Lorenz curve model to present new estimates of income inequality and poverty in rural China. The final section is the conclusion.

2. A new Lorenz curve model

A Lorenz curve is a function relation $L(p)$ which represents the income share earned by the bottom p percent of low income earners. Assume $L(p)$ is defined and continuous in the interval $[0,1]$ with second derivative $L''(p)$. It is called a Lorenz curve if: $L(0) = 0$, $L(1) = 1$, $L'(0^+) \geq 0$ and $L''(p) \geq 0$ for all $p \in [0,1]$.

Sarabia et al. (1999) give two theorems to describe the condition for the basic form of $\tilde{L}(p) = p^\alpha L(p)^\eta$ to be a Lorenz curve which can be briefly written as a single result:

Theorem 1. *If $L(p)$ is a Lorenz curve and $\eta \geq 1$, then*

$$\tilde{L}(p) = p^\alpha L(p)^\eta$$

is a Lorenz curve if one of the following conditions holds

- (1) $\alpha = 0$,
- (2) $\alpha \geq 1$,
- (3) $\alpha \in [0,1]$ and $L''(p) \geq 0$ for all $p \in [0,1]$.

Thus, from a special parametric Lorenz model $L(p)$, three new models can be established. From the Lorenz curve associated with the classical Pareto distribution:

$$S_0(p) = 1 - (1-p)^\beta \tag{1}$$

Sarabia et al. (1999) introduce the generalized Pareto family of Lorenz curves which encompasses the model in Eq. (1) and the following three models:

$$S_1(p) = p^\alpha [1 - (1-p)^\beta], \tag{2}$$

$$S_2(p) = [1 - (1-p)^\beta]^\eta, \tag{3}$$

$$S_3(p) = p^\alpha [1 - (1-p)^\beta]^\eta. \tag{4}$$

S_1 and S_2 are well-known Lorenz models suggested by Ortega et al. (1991) and Rasche et al. (1980) respectively. S_3 is the main model of the family discussed further below. Sarabia et al. (1999) claim that, because $S_0(p)$ is a Lorenz curve with $S_0'''(p) \geq 0$, $S_3(p)$ is a Lorenz curve if Theorem 2 can be satisfied:

Theorem 2. $S_3(p)$ is a Lorenz curve for any

$$\alpha \geq 0, \beta \in (0, 1] \text{ and } \eta \geq 1. \quad (5)$$

The conditions for the models in Eqs. (2) and (3) to be Lorenz curves given by Ortega et al. (1991) and Rasche et al. (1980) respectively are direct corollaries of this theorem. In fact, the condition that $L'''(p) \geq 0$ in theorem 1 above is a severe restriction and it can be relaxed considerably, as the following theorem shows.

Theorem 3. Assume $L(p)$ is a Lorenz curve. $\tilde{L}(p) = p^\alpha L(p)^\eta$ is a Lorenz curve for any $\alpha \geq 0$ and $\eta \geq 1$. Furthermore, if $L'''(p) \geq 0$ for all $p \in [0, 1]$, then $\tilde{L}(p)$ is a Lorenz curve if $\alpha \geq 0$, $\eta \geq 1/2$ and $\alpha + \eta \geq 1$. (For proof see Appendix A).

Ogwang and Rao (2000) suggest two hybrid methods to build Lorenz models. These are the weighted product and the convex combination of Lorenz models. In fact, $\tilde{L}(p)$ belongs to the former category. They find the convex combination model of

$$L_{OR}(p) = \delta p^\alpha [1 - (1-p)^\beta] + (1-\delta) \frac{e^{\lambda p} - 1}{e^\lambda - 1}, \delta \in [0, 1]$$

is satisfactory, where one component is as depicted in Eq. (2) and another is

$$L_C(p) = \frac{e^{\lambda p} - 1}{e^\lambda - 1}, \lambda > 0$$

which is suggested by Chotikapanich (1993). Since both $S_0(p)$, which is associated with the Pareto distribution, and $L_C(p)$ satisfy $L'''(p) \geq 0$ on $[0, 1]$, a different model or a Lorenz curve family can be established by using theorem 3:

$$L_{PC}(p) = p^\alpha \left\{ \delta [1 - (1-p)^\beta] + (1-\delta) \frac{e^{\lambda p} - 1}{e^\lambda - 1} \right\}^\eta$$

with parameter range

$$\alpha \geq 0, \beta \in (0, 1], \lambda > 0, \delta \in [0, 1], \eta \geq 1/2, \alpha + \eta \geq 1.$$

Another application of Theorem 3 is Schader and Schmid's (1994) model:

$$L_{SS}(p) = p^\alpha [1 - \eta p^\gamma (1-p)^\beta].$$

The fitted results of this model are impressive. However, its admissible parameter range cannot be determined so that one cannot be certain if a fitted result satisfies the condition of the Lorenz curve. One option is to replace the term ηp^γ with $e^{-\gamma p}$ in order to create more satisfactory Lorenz models. Using a bi-parametric model:

$$H_0(p) = 1 - (1-p)^\beta e^{-\gamma p}. \quad (6)$$

$H_0(p)$ is a generalization of the classical Pareto Lorenz curve. When it is used as a parametric Lorenz model, we find that it can compete with the models in Eqs. (2) or (3), and, in general, it performs much better than $S_0(p)$.

Following Sarabia et al. (1999) we suggest the following family of Lorenz curves which contains the model in Eq. (6) and the following three models

$$H_1(p) = p^\alpha [1 - (1-p)^\beta e^{-\gamma p}], \quad (7)$$

$$H_2(p) = [1 - (1-p)^\beta e^{-\gamma p}]^\eta, \quad (8)$$

$$H_3(p) = p^\alpha [1 - (1-p)^\beta e^{-\gamma p}]^\eta. \quad (9)$$

Next we address the essential condition for the models to be Lorenz curves. Note $H_i(0) = 0$ and $H_i(1) = 1$ for $i = 0, \dots, 3$ if $\alpha \geq 0$, $\beta > 0$ and $\eta > 0$. For the functions specified in Eqs. (7)–(9) to be Lorenz curves, we need only find the condition for $H_3(p)$ as the other three specifications are special cases of $H_3(p)$.

Theorem 4. $H_3(p)$ is a Lorenz curve if its parameters satisfy either

$$\alpha \geq 0, \beta > 0, 0 \leq \beta + \gamma \leq \sqrt{\beta}, \eta \geq 1 \quad (10)$$

or

$$\alpha \geq 0, \beta \in (0, 1], 0 \leq \beta + \gamma \leq \sqrt{\beta}, \eta \geq 1/2 \text{ and } \alpha + \eta \geq 1. \quad (11)$$

(For a proof of Theorem 4 see Appendix B).

Our estimates suggest that $H_3(p)$ performs better with condition (11) rather than condition (10). Because both $H_0(p)$ and $L_C(p)$ satisfy $L'''(p) \geq 0$, we can give another mixed hybrid family, which is analogous to $L_{PC}(p)$ above, with the main model:

$$L_{HC}(p) = p^\alpha \left\{ \delta \left[1 - (1-p)^\beta e^{-\gamma p} \right] + (1-\delta) \frac{e^{\lambda p} - 1}{e^\lambda - 1} \right\}^\eta,$$

This result can be expressed in the form of the following theorem:

Theorem 5. $L_{HC}(p)$ is a Lorenz curve if

$$\alpha \geq 0, \beta \in (0, 1], 0 \leq \beta + \gamma \leq \sqrt{\beta}, \lambda > 0, \delta \in [0, 1], \eta \geq 1/2 \text{ and } \alpha + \eta \geq 1 \quad (12)$$

3. Indices of income inequality and welfare criteria

Given the density function $f(x)$ of an income distribution, the cumulative distribution function is $F(x) = \int_0^x f(t)dt$, denoting the proportion of those having income less than or equal to x . The Lorenz curve of the distribution is:

$$L(p) = \frac{1}{\mu} \int_0^y x f(x) dx, \quad p = F(y)$$

for any $p \in [0, 1]$, where μ is the mean income of the distribution. This implies that

$$L'(p) = \frac{y}{\mu}, \quad (13)$$

$$L''(p) = \frac{1}{\mu f(y)} \quad (14)$$

Given the Lorenz curve $L(p)$ and income y , we can obtain the associated population share $p = F(y)$ of income earners, whose income is not higher than y , by solving Eq. (13) and the density $f(y)$ given in Eq. (14).

Shorrocks (1983) points out that it is common in practice to rank distributions by comparing the degree of inequality within each distribution first and then comparing mean incomes across the distributions. Assume $L_A(p)$ and $L_B(p)$ are the Lorenz curves of income distribution A and B respectively, and the corresponding mean incomes are μ_A and μ_B respectively. Dasgupta et al. (1973), Rothschild and Stiglitz (1973) and Shorrocks (1983) give a rank dominance criterion when Lorenz curves do not cross: if $\mu_A \geq \mu_B$ and $L_A(p) \geq L_B(p)$ for any $p \in [0, 1]$, then the social welfare of A will be considered to be higher than that of B by any decision maker with a preference for equity. However, this criterion becomes ambiguous and cannot be used when Lorenz curves intersect. To measure social welfare in the case when Lorenz curves intersect Shorrocks (1983) and Kakwani (1984) introduce the generalized

Table 1

Parameter estimates of model L_{HC} for rural China 1980–2006.

	α	β	γ	λ	η	δ
1980	0.000000 (0.306095)	0.827299 (0.000295)	−0.151801 (0.071939)	16.903717 (5.736738)	1.174113 (0.312405)	0.986543 (0.015802)
1985	0.376926 (0.041535)	0.712083 (0.000051)	−0.107402 (0.021646)	15.922273 (2.936712)	0.858265 (0.045294)	0.987997 (0.003648)
1990	0.000000 (0.063856)	0.767595 (0.000093)	−0.108198 (0.016266)	17.394979 (0.825867)	1.240938 (0.066570)	0.986175 (0.003666)
1995	0.000000 (0.109455)	0.713369 (0.000168)	−0.036759 (0.045076)	4.717662 (3.176295)	1.319719 (0.117831)	0.977824 (0.017570)
2000	0.824899 (0.033063)	0.564659 (0.001124)	−0.231362 (0.025680)	10.678289 (0.432160)	0.520617 (0.035878)	0.942833 (0.014020)
2001	0.565822 (0.030050)	0.621467 (0.000498)	−0.146301 (0.013523)	12.499393 (0.427549)	0.781345 (0.031334)	0.969952 (0.005779)
2002	0.783171 (0.031204)	0.549674 (0.001279)	−0.217789 (0.020916)	10.721184 (0.286933)	0.553394 (0.033273)	0.941961 (0.012967)
2003	0.514661 (0.027175)	0.624085 (0.000641)	−0.155918 (0.011891)	15.230320 (1.170289)	0.833702 (0.028453)	0.970886 (0.004152)
2004	0.857333 (0.098066)	0.527749 (0.006400)	−0.206886 (0.064551)	9.531963 (0.667123)	0.500000 (0.102516)	0.924452 (0.050691)
2005	0.624236 (0.092610)	0.597189 (0.003227)	−0.152793 (0.038056)	9.831746 (1.615751)	0.734106 (0.094428)	0.965575 (0.022617)
2006	0.876126 (0.086525)	0.511258 (0.008059)	−0.182163 (0.047759)	6.509688 (0.531540)	0.500000 (0.088139)	0.909659 (0.049658)

Note: standard errors in parentheses.

Table 2

Data and Lorenz curve for rural China 1980–2006.

x_i	p_i	$L(p_i)$	$\hat{L}_{HC}(p_i)$	x_i/μ	$\hat{L}'_{HC}(p_i)$
1980					
60	0.00908	0.00249	0.00249	0.31250	0.32327
80	0.03858	0.01366	0.01376	0.41667	0.42333
100	0.10223	0.04410	0.04412	0.52084	0.52138
150	0.37888	0.22622	0.22609	0.78126	0.78322
200	0.63553	0.45882	0.45900	1.04168	1.04493
250	0.80067	0.65097	0.65082	1.30210	1.30102
300	0.89253	0.78106	0.78112	1.56252	1.56615
500	0.98818	0.96305	0.96302	2.60420	2.59522
1985					
100	0.01014	0.00191	0.00222	0.26115	0.27086
150	0.04661	0.01461	0.01473	0.39172	0.39416
200	0.13289	0.05512	0.05494	0.52229	0.52500
300	0.40249	0.23282	0.23291	0.78344	0.78564
400	0.64689	0.45474	0.45471	1.04459	1.04340
500	0.80193	0.63459	0.63455	1.30573	1.30079
600	0.88712	0.75525	0.75535	1.56688	1.56379
800	0.95733	0.88004	0.87995	2.08917	2.08181
1000	0.98165	0.93586	0.93588	2.61146	2.60359
1500	0.99604	0.97935	0.97950	3.91720	3.83269
2000	0.99856	0.99043	0.99026	5.22293	4.95706
1990					
100	0.00218	0.00021	0.00029	0.14577	0.16640
150	0.00737	0.00122	0.00133	0.21865	0.22382
200	0.02115	0.00486	0.00493	0.29153	0.29076
300	0.09519	0.03267	0.03264	0.43730	0.43609
400	0.22731	0.10044	0.10047	0.58306	0.58304
500	0.37689	0.19858	0.19851	0.72883	0.72783
600	0.51839	0.31164	0.31168	0.87459	0.87525
800	0.72580	0.52083	0.52085	1.16612	1.16479
1000	0.84525	0.67569	0.67566	1.45765	1.45837
1500	0.95513	0.86721	0.86724	2.18648	2.20427
2000	0.98421	0.93990	0.93987	2.91531	2.92553
1995					
200	0.00418	0.00040	0.00042	0.12651	0.13331
300	0.01324	0.00189	0.00194	0.18977	0.19350
400	0.03065	0.00578	0.00589	0.25302	0.25505
500	0.05686	0.01327	0.01340	0.31628	0.31454
600	0.09526	0.02665	0.02675	0.37953	0.37777
800	0.19970	0.07320	0.07315	0.50605	0.50465
1000	0.32442	0.14438	0.14417	0.63256	0.63295
1200	0.44600	0.22875	0.22876	0.75907	0.76035
1300	0.50009	0.27146	0.27154	0.82232	0.82216
1500	0.59801	0.35798	0.35807	0.94883	0.94935
1700	0.67587	0.43658	0.43664	1.07535	1.07325
2000	0.76608	0.54153	0.54151	1.26511	1.26254
2500	0.86264	0.67765	0.67738	1.58139	1.58291
3000	0.91659	0.77058	0.77058	1.89767	1.90212
3500	0.94778	0.83418	0.83448	2.21395	2.22493
4000	0.96433	0.87321	0.87349	2.53023	2.50940
4500	0.97541	0.90287	0.90283	2.84650	2.80997
5000	0.98233	0.92361	0.92320	3.16278	3.09953
2000					
200	0.00225	0.00015	0.00015	0.08825	0.08973
300	0.00740	0.00073	0.00075	0.13238	0.13580
400	0.01566	0.00202	0.00205	0.17651	0.17685
500	0.02726	0.00434	0.00434	0.22064	0.21576
600	0.04333	0.00826	0.00814	0.26476	0.25577
800	0.09436	0.02366	0.02363	0.35302	0.34554
1000	0.15872	0.04863	0.04871	0.44127	0.43088
1200	0.23305	0.08393	0.08397	0.52952	0.51653
1300	0.27345	0.10571	0.10573	0.57365	0.56087
1500	0.35339	0.15415	0.15404	0.66191	0.64782
1700	0.43218	0.20851	0.20853	0.75016	0.73597
2000	0.53726	0.29245	0.29244	0.88254	0.86399

Table 2 (continued)

x_i	p_i	$L(p_i)$	$\hat{L}_{HC}(p_i)$	x_i/μ	$\hat{L}'_{HC}(p_i)$
2000					
2500	0.67998	0.43029	0.43035	1.10318	1.08038
3000	0.77851	0.54681	0.54674	1.32381	1.29637
3500	0.84490	0.63958	0.63958	1.54445	1.51532
4000	0.88820	0.70944	0.70949	1.76508	1.72681
4500	0.91867	0.76521	0.76518	1.98572	1.94137
5000	0.93936	0.80735	0.80735	2.20635	2.14739
2001					
200	0.00222	0.00014	0.00015	0.08219	0.08829
300	0.00601	0.00053	0.00056	0.12328	0.12498
400	0.01299	0.00155	0.00158	0.16438	0.16401
500	0.02380	0.00357	0.00358	0.20547	0.20364
600	0.03986	0.00722	0.00720	0.24656	0.24582
800	0.08492	0.02030	0.02026	0.32875	0.32814
1000	0.14712	0.04337	0.04338	0.41094	0.41220
1200	0.21663	0.07482	0.07486	0.49313	0.49228
1300	0.25453	0.09431	0.09430	0.53422	0.53347
1500	0.33224	0.13901	0.13899	0.61641	0.61653
1700	0.40802	0.18881	0.18882	0.69860	0.69913
2000	0.51146	0.26730	0.26730	0.82188	0.82073
2500	0.65620	0.40040	0.40040	1.02735	1.02876
3000	0.75667	0.51328	0.51331	1.23282	1.23165
3500	0.82605	0.60548	0.60546	1.43829	1.43841
4000	0.87242	0.67662	0.67660	1.64376	1.64255
4500	0.90492	0.73308	0.73312	1.84923	1.84870
5000	0.92721	0.77645	0.77644	2.05470	2.04651
2002					
200	0.00200	0.00012	0.00013	0.07818	0.08689
300	0.00517	0.00044	0.00046	0.11727	0.11976
400	0.01101	0.00125	0.00127	0.15636	0.15508
500	0.02038	0.00291	0.00291	0.19545	0.19193
600	0.03532	0.00613	0.00610	0.23454	0.23320
800	0.07771	0.01781	0.01779	0.31272	0.31258
1000	0.13446	0.03783	0.03781	0.39090	0.39025
1200	0.20315	0.06738	0.06740	0.46908	0.46967
1300	0.23852	0.08467	0.08469	0.50817	0.50800
1500	0.31216	0.12497	0.12498	0.58634	0.58606
1700	0.38547	0.17081	0.17081	0.66452	0.66461
2000	0.48732	0.24438	0.24433	0.78179	0.78101
2500	0.63122	0.37023	0.37026	0.97724	0.97852
3000	0.73299	0.47901	0.47905	1.17269	1.17044
3500	0.80468	0.56955	0.56949	1.36814	1.36453
4000	0.85631	0.64485	0.64484	1.56359	1.56661
4500	0.89109	0.70243	0.70249	1.75903	1.75896
5000	0.91598	0.74853	0.74850	1.95448	1.94711
2003					
200	0.00194	0.00011	0.00011	0.07387	0.07949
300	0.00544	0.00044	0.00046	0.11081	0.11404
400	0.01144	0.00122	0.00125	0.14774	0.14829
500	0.02063	0.00276	0.00279	0.18468	0.18315
600	0.03436	0.00557	0.00557	0.22161	0.22068
800	0.07265	0.01554	0.01551	0.29549	0.29357
1000	0.12763	0.03385	0.03384	0.36936	0.37036
1200	0.18995	0.05925	0.05925	0.44323	0.44366
1300	0.22275	0.07440	0.07440	0.48016	0.47977
1500	0.29176	0.11003	0.11007	0.55403	0.55378
1700	0.35983	0.15028	0.15025	0.62791	0.62710
2000	0.45893	0.21791	0.21790	0.73871	0.73976
2500	0.59769	0.33282	0.33284	0.92339	0.92392
3000	0.70382	0.44017	0.44016	1.10807	1.10782
3500	0.78059	0.53184	0.53186	1.29275	1.29174
4000	0.83505	0.60698	0.60697	1.47743	1.47728
4500	0.87319	0.66661	0.66662	1.66210	1.66110
5000	0.90111	0.71547	0.71546	1.84678	1.84730
2004					
200	0.00134	0.00007	0.00007	0.06626	0.06968
300	0.00352	0.00025	0.00025	0.09939	0.09837

(continued on next page)

Table 2 (continued)

x_i	p_i	$L(p_i)$	$\hat{L}_{HC}(p_i)$	x_i/μ	$\hat{L}'_{HC}(p_i)$
2004					
400	0.00695	0.00065	0.00064	0.13252	0.12575
500	0.01322	0.00159	0.00154	0.16566	0.15877
600	0.02327	0.00344	0.00333	0.19879	0.19546
800	0.05200	0.01016	0.01002	0.26505	0.26491
1000	0.09413	0.02274	0.02275	0.33131	0.33571
1200	0.13991	0.03951	0.03956	0.39757	0.39750
1300	0.16773	0.05104	0.05110	0.43070	0.43134
1500	0.22436	0.07732	0.07737	0.49697	0.49569
1700	0.28603	0.11002	0.11000	0.56323	0.56242
2000	0.37890	0.16691	0.16686	0.66262	0.66230
2500	0.52052	0.27208	0.27204	0.82828	0.82712
3000	0.63928	0.37987	0.37990	0.99393	0.99654
3500	0.72429	0.47103	0.47108	1.15959	1.15643
4000	0.79065	0.55320	0.55315	1.32524	1.32590
4500	0.83849	0.62037	0.62037	1.49090	1.49253
5000	0.87422	0.67651	0.67652	1.65656	1.65924
2005					
200	0.00101	0.00005	0.00005	0.05943	0.06170
300	0.00313	0.00021	0.00021	0.08914	0.09264
400	0.00632	0.00054	0.00055	0.11885	0.11931
500	0.01095	0.00116	0.00117	0.14857	0.14572
600	0.01806	0.00232	0.00232	0.17828	0.17503
800	0.04056	0.00704	0.00701	0.23771	0.23697
1000	0.07303	0.01575	0.01574	0.29714	0.29820
1200	0.11275	0.02877	0.02879	0.35656	0.35680
1300	0.13496	0.03702	0.03704	0.38628	0.38571
1500	0.18490	0.05782	0.05781	0.44571	0.44536
1700	0.23809	0.08310	0.08309	0.50513	0.50451
2000	0.32168	0.12906	0.12903	0.59427	0.59450
2500	0.45429	0.21749	0.21753	0.74284	0.74255
3000	0.56961	0.31146	0.31143	0.89141	0.89039
3500	0.66436	0.40262	0.40261	1.03998	1.04016
4000	0.73748	0.48384	0.48387	1.18855	1.18897
4500	0.79280	0.55355	0.55353	1.33712	1.33628
5000	0.83593	0.61426	0.61427	1.48569	1.48705
2006					
200	0.00104	0.00004	0.00004	0.05430	0.05702
300	0.00218	0.00012	0.00012	0.08145	0.07536
400	0.00553	0.00044	0.00043	0.10860	0.10710
500	0.01010	0.00100	0.00099	0.13574	0.13463
600	0.01581	0.00186	0.00183	0.16289	0.15982
800	0.03487	0.00550	0.00547	0.21719	0.21743
1000	0.06169	0.01209	0.01209	0.27149	0.27347
1200	0.09366	0.02167	0.02169	0.32579	0.32568
1300	0.11232	0.02800	0.02802	0.35294	0.35239
1500	0.15456	0.04408	0.04408	0.40723	0.40706
1700	0.20146	0.06449	0.06449	0.46153	0.46230
2000	0.27462	0.10130	0.10130	0.54298	0.54334
2500	0.39734	0.17621	0.17620	0.67872	0.67825
3000	0.51200	0.26168	0.26170	0.81447	0.81641
3500	0.60531	0.34388	0.34387	0.95021	0.94903
4000	0.68373	0.42347	0.42347	1.08596	1.08579
4500	0.74597	0.49511	0.49511	1.22170	1.22166
5000	0.79501	0.55819	0.55819	1.35745	1.35594

Notes: the average income μ for 1980, 1985, 1990, 1995, 2000–2006 are respectively: 192.00, 382.93, 686.03, 1580.89, 2433.44, 2558.22, 2707.41, 3018.31, 3365.45, 3683.39.

Lorenz (GL) curve, which is defined as $GL(p) = \mu L(p)$. The rank dominance criterion in terms of GL is that any decision maker with a preference for both equity and efficiency (the higher the income the better) prefers A to B , if and only if $\mu_{AL_A}(p) \geq \mu_{BL_B}(p)$ for all $p \in [0,1]$, meaning that the two GL curves do not intersect. A major advantage of welfare comparison in terms of GL curves is that there may be more distributions that can be compared than with Lorenz curves, since differences between Lorenz curves tend to be relatively small compared with variations in mean incomes, as noted by Shorrocks (1983). There may be many cases where mean income is high enough to offset the lower part of intersecting Lorenz curves to generate GL curves which do not intersect. There are some studies that further develop welfare comparison methods when Lorenz curve or GL curves intersect finite times (see eg. Davies & Hoy, 1995; Lambert, 1989). However, in these cases, the rank issue becomes complicated. Thus, normally, it is necessary to rely on inequality indices to compare distributions when GL curves intersect.

The head-count ratio is the proportion of the population whose income falls below the poverty line. For a given poverty line $y = z$, the head-count ratio is $H = F(z)$, obtained by solving $\mu L'(p) = z$. The Lorenz curve of the poor is:

$$L_p(p) = \frac{1}{\mu_p H} \int_0^y x f(x) dx = \frac{L(pH)}{L(H)}, \quad p = F(y) / H \text{ and } y \in [0, z],$$

where μ_p is the average income of the poor,

$$\mu_p = \frac{1}{H} \int_0^z x f(x) dx = \frac{\mu L(H)}{H},$$

and the Gini index among the poor is

$$G_p = 1 - \frac{2}{L(H)H} \int_0^H L(p) dp.$$

Apart from the head-count ratio H , we will use the following two well-known poverty indices to measure poverty. Let $I = (z - \mu_p) / z$. The first index is

$$S = H(I + (1 - I)G_p) = H - \frac{2\mu}{zH} \int_0^H L(p) dp \quad (15)$$

proposed by Sen (1976). Another well-known poverty index is

$$F_\alpha = \int_0^z \left(\frac{z-x}{z} \right)^\alpha f(x) dx = \int_0^H \left(\frac{z - \mu L'(p)}{z} \right)^\alpha dp, \quad \alpha > 0 \quad (16)$$

proposed by Foster, Greer, and Thorbecke (1984). Foster and Shorrocks (1991) note that F_α has several attractive properties. To measure income inequality we use the Gini coefficient:

$$G = 2 \int_0^1 (p - L(p)) dp$$

and the index proposed by Kakwani (1980):

$$K_k = 1 - k(k+1) \int_0^1 L(p)(1-p)^{k-1} dp, \quad k \geq 1 \quad (17)$$

which is equal to G when $k = 1$. Chakravarty (1988) suggests using:

$$C_\alpha = 2 \left(\int_0^1 (p - L(p))^\alpha dp \right)^{1/\alpha}, \quad \alpha > 1. \quad (18)$$

This index satisfies the diminishing transfers axiom (see Chakravarty, 1988).

When grouped data $\{(p_i, L_i)\}_{i=1}^m$ is the only accessible data on the Lorenz curve of the underlying distribution, where p_i is the share of low income earners and L_i the income share of the low income population, a Lorenz model $L(p, \tau)$ can be selected to fit the data. Let τ be the parameter vector of the model which is estimated by minimizing the sum of residual squares (SRS)

$$\sum_{i=1}^m (L_i - L(p_i, \tau))^2.$$

Assume the estimation of τ is $\hat{\tau}$. Not only can the estimated Lorenz curve $L(p, \hat{\tau})$ be used to approximate the Lorenz curve $L(p)$ in calculating the above indices, but an approximation density $\hat{f}(y)$ can be obtained by (13) and (14) from $L(p, \hat{\tau})$. $\hat{f}(y)$ is the best possible estimate in the sense that $L(p, \hat{\tau})$, which is the Lorenz curve associated with $\hat{f}(y)$, is optimal in terms of the minimization criterion of SRS. Furthermore, we can obtain the associated estimated cumulative distribution function $\hat{F}(y)$ by either $\hat{F}(y) = \int_0^y \hat{f}(x) dx$ or by solving $\mu L'(p; \hat{\tau}) = y$ for any given y . If income intervals $(x_{i-1}, x_i]$, $i = 1, 2, \dots$, are also given such that the related population share of income earners, whose income is not higher than x_i , is p_i as given in $\{(p_i, L_i)\}_{i=1}^m$, $\hat{F}(x_i)$ is then the estimation of p_i .

4. Measuring income inequality and poverty in rural China

4.1. The data

We use grouped data on net income compiled by the State Statistical Bureau (SSB) for the years 1980, 1985, 1990, 1995, 2001–2006 as published in the China Rural Household Survey Yearbook (SSB, 2001–2007). The data contain the group interval $(x_{i-1}, x_i]$, $i = 1, 2, \dots$,

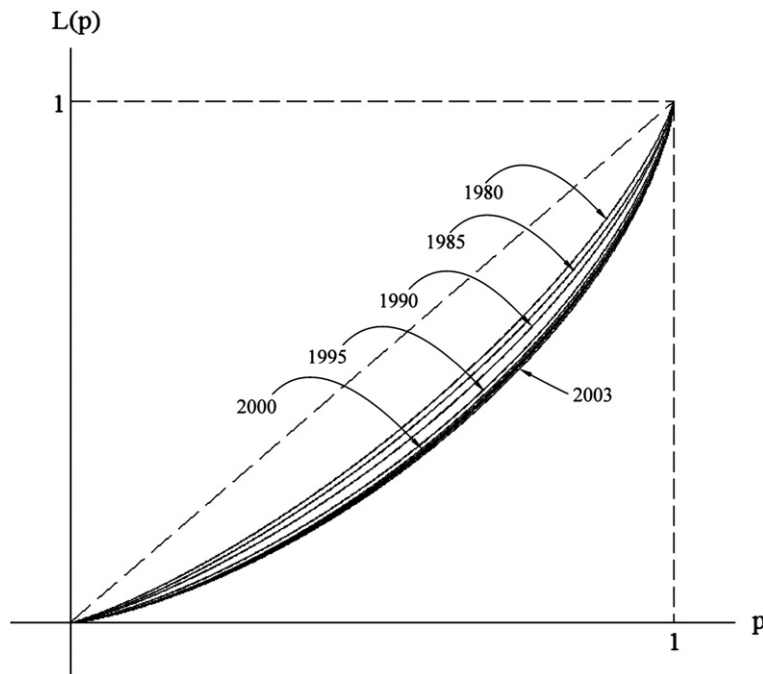


Fig. 1. Lorenz curves for rural China 1980 to 2006.

frequency of households f_i in the interval, average number of persons n_i in each household and average annual income per capita μ_i in the interval. The Rural Household Survey (RHS) is organized and conducted by the Rural Society and Economy Investigation Department of the SSB. Chen and Ravallion (1996) give details of the Survey. In brief, about 8000 SSB staff together with about 10000 assistants are involved in collecting the data. As of 2007, the RHS contained a sample of some 68000 households from 860 counties. The latter are selected from 2450 counties in the entire country. The SSB has a sample replacement mechanism for updating the sample annually. It rotates 20% of households to prevent sample aging or attrition, replacing the sample completely each five years. The counties, however, are never rotated for administration reasons. The sampled households are required to report their production, income and expenditure in detail throughout the year. While SSB has released some parts of the RHS to a few scholars for research purposes in the past, the entire dataset is unavailable outside the SSB. The data in grouped form is the result of aggregating from the RHS.

4.2. Lorenz curves and welfare comparisons

Table 1 contains the parameter estimates for model L_{HC} , where the corresponding estimated curve $\hat{L}_{HC}(p)$ satisfies the definition of the Lorenz curve. The sample and estimated Lorenz values are listed in Table 2, where the second and third columns are empirical Lorenz values $(p_i, L(p_i))_{i=1}^n$. The estimates for $L(p_i)$, listed in the column titled $\hat{L}_{HC}(p_i)$, are very close to $L(p_i)$. Let the mean income be μ . Note the derivative of the underlying Lorenz curve $L(p)$ at $p = p_i$ is $L'(p_i) = x_i/\mu$, which can be obtained from the original data, where $p_i = F(x_i)$. From Table 2 it can be seen that the estimated derivative $\hat{L}'_{HC}(p)$ is very close to x_i/μ , which also provides evidence that L_{HC} is a very good model for the data to which it is applied.

The Lorenz curve estimates are in Fig. 1. Let the Lorenz curve for year i be $L_i(p)$. We find that $L_i(p) \geq L_j(p)$ for any $p \in [0,1]$ for $i < j < 2001$, as can be seen in Fig. 1. Therefore, the distribution with $L_i(p)$ is unambiguously more equal than the distribution with $L_j(p)$. $L_i(p) \geq L_j(p)$ also holds in the upper part of the Lorenz curves for $2001 \leq i < j$. But, there are some points near $p = 0$ where $L_{2000}(p) < L_{2002}(p)$, $L_{2001}(p) < L_{2002}(p)$, $L_{2003}(p) < L_{2004}(p)$ and $L_{2003}(p) < L_{2005}(p)$, indicating that these pairs of Lorenz curves intersect with each other at least once. Thus, we do not know the complete inequality ordering across all years by Lorenz curve comparisons. However, Fig. 1 clearly suggests that the income share of the population at the lower part of the distribution is getting smaller. We examine the Lorenz curves for 11 years over the period 1980 to 2006 at population shares $p = 0.3, 0.2, 0.1, 0.05$ and list the values in Table 3. It can be seen, for

Table 3
Income shares at the lower part of the distributions for rural China 1980 to 2006.

P	1980	1985	1990	1995	2000	2001	2002	2003	2004	2005	2006
0.30	0.1671	0.1572	0.1454	0.1290	0.1210	0.1197	0.1179	0.1147	0.1180	0.1164	0.1154
0.20	0.1003	0.0927	0.0849	0.0733	0.0675	0.0668	0.0659	0.0638	0.0656	0.0647	0.0638
0.10	0.0430	0.0383	0.0348	0.0285	0.0256	0.0254	0.0251	0.0241	0.0247	0.0244	0.0238
0.05	0.0187	0.0161	0.0145	0.0113	0.0099	0.0098	0.0098	0.0093	0.0095	0.0093	0.0090

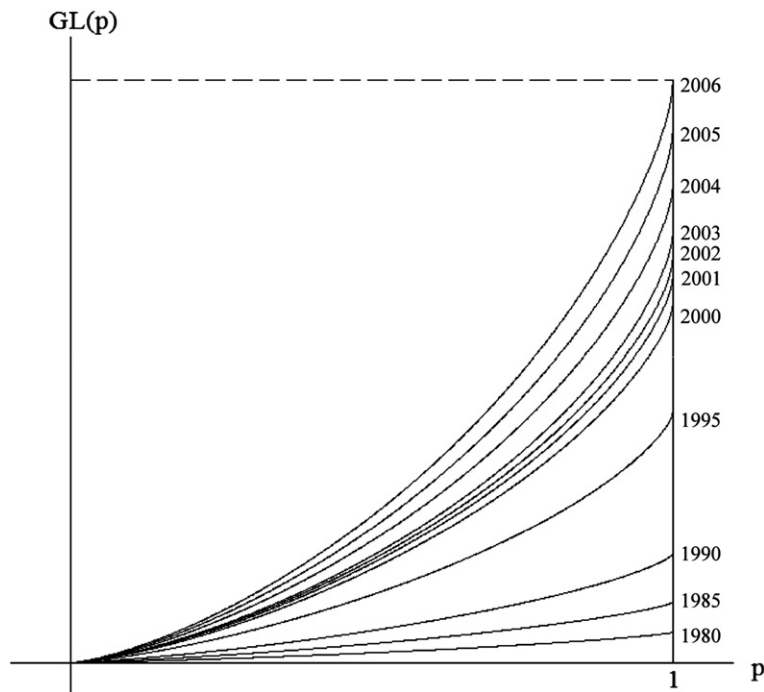


Fig. 2. Generalized Lorenz curves for rural China for 11 sample years between 1980 and 2006.

example, that the income share of low income earners with population share $p = 30\%$ decreased from 16.71% in 1980 to 11.54% in 2006. The income share of low income earners with population share $p = 5\%$ decreased from 1.87% in 1980 to 0.9% in 2006, meaning that their share has more than halved. This implies that the rural poor have not been able to catch up and share the benefits of economic reform.

Since we cannot make welfare comparisons in terms of Lorenz curves for some of the 11 years in the sample, we make welfare comparison in terms of GL curves. Let $GL_i(p)$ be the GL curve of year i . Denote $GL_i \geq_G GL_j$ if $GL_i(p) \geq GL_j(p)$ for all $p \in [0,1]$. We find that $GL_i \geq_G GL_j$ generally holds if $i > j$, except in the lower part of the distributions where we find $GL_{2003}(p) < GL_{2002}(p)$ and $GL_{2005}(p) <$

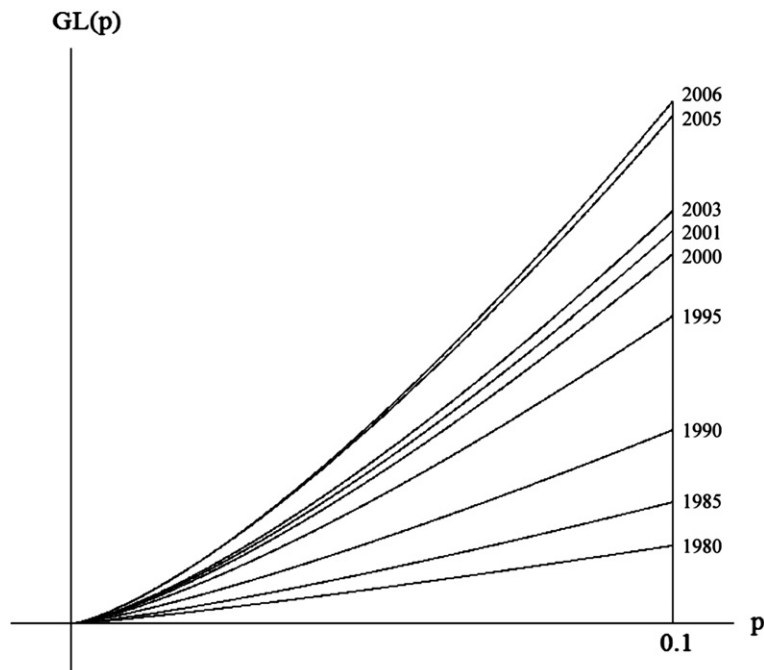


Fig. 3. Lower part of generalized Lorenz curves for rural China for nine years 1980–2006.

Table 4

Inequality indices for rural China 1980 to 2006.

	K_3	C_2	G	GL	GU	SSB Gini
1980	0.39781	0.26863	0.24950	0.24038	0.25423	0.2407
1985	0.42970	0.29392	0.27371	0.26604	0.27771	0.2267
1990	0.46561	0.32323	0.30026	0.29431	0.30330	0.3099
1995	0.51687	0.36414	0.33810	0.33561	0.33931	0.3415
2000	0.54015	0.38146	0.35451	0.35025	0.35659	0.3536
2001	0.54532	0.38818	0.36057	0.35549	0.36372	0.3603
2002	0.55136	0.39546	0.36727	0.36100	0.37104	0.3646
2003	0.56135	0.40382	0.37480	0.36729	0.37990	0.3680
2004	0.55084	0.39309	0.36524	0.35500	0.37193	0.3692
2005	0.55558	0.39702	0.36869	0.35378	0.37922	0.3751
2006	0.55793	0.39693	0.36882	0.34791	0.38330	0.3737

Notes:

 K_3 : Kakwani index. C_2 : Chakravarty index. G : Estimated Gini index. GL : Gini lower bound. GU : Gini upper bound.

SSB Gini: SSB Gini index.

$GL_{2004}(p)$ at some points near $p = 0$. Thus $GL_{2004} \geq_G GL_{2003}$, $GL_{2002} \geq_G GL_{2001}$ and $GL_{2006} \geq_G GL_{2005} \geq_G GL_{2003} \geq_G GL_{2001} \geq_{G'''} \geq_G GL_{1980}$. Thus, apart from two years, social welfare is also increasing in terms of the welfare criterion of GL curves. To visualize the GL dominance, Fig. 2 gives the GL curves and Fig. 3 depicts the lower part of the GL curves, excluding 2002 and 2004.

4.3. Inequality and poverty estimates

Table 4 presents the inequality indices for rural China for the period 1980 to 2006. Examining the inequality indices in Table 4, the Gini index has increased almost 50% over the period 1980 to 2006, from a quite low level of 0.25 to about 0.37. The Chakravarty and Kakwani indices reported in Table 4 also exhibit a rapid increase. Gastwirth (1972) derives the lower bound GL and upper bound GU for Gini indices from grouped data. Some authors use these bounds to test the goodness of fit of a Lorenz model for grouped data (see, for example, Kakwani, 1980; Schader & Schmid, 1994). Schader and Schmid (1994) find that many traditional Lorenz models may produce Gini estimates which extend outside the Gastwirth bounds. We estimate the GL and GU for each year and present them in Table 4. It can be found that our estimated Gini indices G are all between GL and GU. The last column of Table 4 contains the Gini indices given by the SSB. We find our estimated Gini index for each year is very close to the corresponding SSB Gini, except for the Gini for 1985.

The increase in income inequality evident in Table 4 can be divided into three phrases from the inequality indices. The first is 1980–1995 when the rate of increase in income inequality was the fastest. Several explanations have been offered for rising income inequality over this period. Some have suggested that the emergence of the non-agricultural sector in the 1980s and first half of the 1990s, particularly the collective township and village enterprise (CTVE, *xiang-zhen qiye*) sector, changed the composition of rural income and generated higher inequality (Hare, 1994; Khan & Riskin, 1998; Kung & Lee, 2001; Rozelle, 1994, 1996; Tsui, 1998; Zhang, 1992; Zhu, 1992). Decollectivization gave rural households more discretion in their production decisions. With this new found freedom and the small land-to-person ratio available in many rural areas, it was natural for rural labor to move into CTVEs (Tsui, 1998). The emergence of CTVEs was also related to fiscal decentralization. Fiscal decentralization placed pressure on local governments to raise revenue and sub-provincial governments invested in CTVEs, the taxes from which became an important source of revenue (Zhang, 2006). Some have also linked the disequalizing effect of non-agricultural activities on the concentration

Table 5Poverty indices for rural China 1980 to 2006 (z : SSB's absolute poverty line).

	z	S	F_2	H	Poor population (million)
1980	130.00	0.07589	0.01713	0.25959	213.10
1985	206.00	0.04395	0.01052	0.14408	121.69
1990	300.00	0.02931	0.05433	0.09607	86.88
1995	530.00	0.02454	0.00688	0.06828	62.89
2000	625.00	0.02006	0.00590	0.05297	51.26
2001	630.00	0.01729	0.00507	0.04582	41.91
2002	627.00	0.01498	0.00431	0.04054	38.10
2003	637.00	0.01543	0.00454	0.04081	38.18
2004	668.00	0.01239	0.00368	0.03246	30.25
2005	683.00	0.01026	0.00305	0.02691	25.45
2006	693.00	0.00947	0.00289	0.02410	22.51

Table 6

Poverty indices for rural China 1980 to 2006 (z: SSB low-income line).

	<i>z</i>	<i>S</i>	<i>F</i> ₂	<i>H</i>	Poor population (million)
1980	235.12	0.35199	0.12265	0.76056	624.34
1985	266.03	0.10303	0.02699	0.30305	255.95
1990	439.22	0.10157	0.12415	0.28620	258.80
1995	775.23	0.07127	0.02101	0.18652	171.80
2000	835.35	0.04329	0.01310	0.11046	106.89
2001	842.00	0.03765	0.01131	0.09693	88.66
2002	869.00	0.03678	0.01090	0.09602	90.26
2003	882.00	0.03677	0.01111	0.09410	88.03
2004	924.00	0.02934	0.00888	0.07505	69.96
2005	944.00	0.02437	0.00735	0.06263	59.25
2006	958.00	0.02177	0.00672	0.05456	50.95

Note: the low-income line for 1980–2000 was obtained by deflating, using the rural CPI.

of such activities in the rich coastal seaboard provinces with restrictions on interregional factor mobility (Rozelle, 1994; Tsui, 1998; Zhang, 1992; Zhu, 1992). Others focus on the lack of an equitable taxation system and the imposition of arbitrary fees and levies at the local level as a reason for rural income inequalities (Wong, Heady, & West, 1995). Yet others have emphasized the role played by political connections in opening up new opportunities for cadre-entrepreneurs and associates (Benjamin, Brandt, Glewwe, & Li, 1992; Cook, 1998; Nee, 1992).

The second phase is 1995–2003, when the rate of increase in income inequality was much smaller. The level of income inequality stabilized after 2003, which constitutes the third phrase. Chotikapanich et al. (2007) also find that rural income inequality starts to stabilize from the mid-1990s following a rapid increase in rural income inequality in the 1980s. One possible explanation for the slower increase in rural inequality from the mid-1990s was the introduction of preferential policies in 1993 to promote CTVEs in the central and western regions of China which was aimed at increasing non-farm income in rural areas in poorer provinces (Tsui, 1998). There is evidence that provincial rural income differences have declined over time (Benjamin et al., 2005). Another possible explanation is increased access of the rural poor to non-farm income from the mid-1990s. While several studies cited above suggest that non-farm income increases rural income inequality, some studies using data from the mid-1990s onwards have found that participation rates in non-farm activity among low-income rural households increased from the mid-1990s and this resulted in a more equal distribution of income (Fan, Zhang, & Zhang, 2002; Zhu & Luo, 2006). Zhu and Luo (2006) study the distribution of non-farm income in rural Hebei and Liaoning over the period 1995 to 1997. Their finding is that non-farm activity participation reduced rural income inequality by widening the occupation choice that disproportionately favors poor households. The decline in the importance of restrictions on migration created by China's residential system (*hukou* system) and the opening up of markets for migrant labor theoretically should also have contributed to a decline in the importance of regional differences in overall inequality (Benjamin et al., 2005; Tsui, 1998). The available evidence on the effect of migration on rural inequality, although sketchy, suggests families in rural areas receiving remittances are not the poorer households so migration might actually exacerbate rural income inequalities (Tsui, 1998).

We estimated poverty rates using two sets of poverty lines. The first set z_1 is the 'absolute poverty line' and the second and higher set z_2 is the 'low-income line'. Both of the lines are provided by the SSB. The variation in lines reflects fluctuation in rural

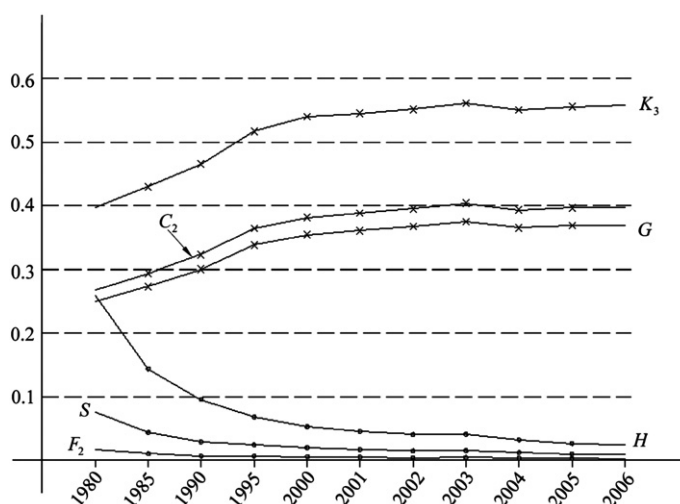


Fig. 4. Inequality and poverty indices based on SSB absolute poverty line.

which was implemented from 1993–2000. The goal of this program was to reduce the proportion of the population that is poor by 80 million in the last seven years of the twentieth century.

Figs. 4 and 5 allow us to compare variations in poverty and inequality indices over time. Fig. 4 compares trends in income inequality with trends in absolute poverty and Fig. 5 compares trends in income inequality with trends in the SSB low income line. In both Figs. 4 and 5 the inequality indices from Table 4 are depicted by the three polygonal lines at the top of the figure corresponding to K_3 , C_2 and G . The three polygonal lines at the bottom of Fig. 4 depict the poverty indices using the absolute poverty line from Table 5, while the three polygonal lines at the bottom of Fig. 5 depict the poverty indices using the SSB low income line from Table 6.

Three observations can be made about Fig. 4. First, the three phrases of variation in rural income inequality as described above are clearly seen. There is a steep increase in income inequality over the period 1980 to 1995; the inequality indices start to flatten out after 1995 and are flat from 2003 onwards. Second, the increases in the income inequality indices over time correspond closely to decreases in the poverty indices calculated using the absolute poverty line. The poverty indices decline sharply over the period 1980 to 1995 and then flatten out considerably after 1995. Third, the marginal reduction of poverty over time has been decreasing. This is consistent with the existence of relatively large one off gains in the 1980s made possible by undoing the failed policies of the Cultural Revolution with subsequent gains more difficult. In comparison with Figs. 4 and 5 reveals more clearly the trend that increases in income inequality indices over time correspond to decreases in poverty indices and that the marginal reduction of the poverty indices is decreasing. The much larger scale variation of poverty indices over time reflects more clearly the success of poverty reduction in rural China attributed to the economic reform. That the marginal reduction in poverty indices has decreased over time is consistent with other research suggesting that the decline in poverty in rural China slowed dramatically, and may have even reversed, during the 1990s (Chen & Ravallion, 2004; Zhang & Wan, 2006).

The Chinese government has allocated a lot of funds to support the national designated poverty counties each year since 1986 (see Park, Wang, & Wu, 2002). However, as Figs. 4 and 5 show, the marginal reduction in poverty is diminishing. Some authors suggest that the rural poor in China have been marginalized both in the geographic and demographic sense, meaning that the remaining rural poor mainly consist of people with adverse circumstances, such as low educational endowment or bad health or are those who reside in regions with harsh natural conditions (Cai & Du, 2006). This argument clearly implies that from the viewpoint of reducing poverty in rural China some re-distributional instrument, for example, special transfers from the government, should have been employed from the beginning of the reforms. In this respect, comparing the trajectories of growth in GDP and poverty reduction since the late 1980s with earlier periods, several studies have emphasized the role of rural income inequality in slowing poverty reduction in China (Chen & Wang, 2001; Gustafsson & Wei, 2000; Ravallion & Chen, 2007; Yao, 1999; Zhang & Wan, 2006).

Of course suggesting that some re-distributional instrument should have been employed from the start of the reforms ignores the important question: Could the reforms have been as successful as they have been in terms of increasing mean incomes if a stronger re-distributive mechanism had been in place? There is a voluminous theoretical and empirical literature on the effects of income inequality on growth. From a theoretical perspective there are arguments going both ways (Wan, Lu, & Chen, 2006). Several theoretical arguments suggest that inequality should have a negative effect on economic growth. First, rising income inequality generates an increase in the fertility rate among the poor and lowers investment in human capital in the poor (De La Croix & Doepke, 2004). Second, given the prevalence of imperfect capital markets, rising income inequality increases the credit constraints facing individuals and lowers their ability to invest in human and physical capital (Fishman & Simhon, 2002). Third, rising income inequality might weaken domestic demand which slows growth (Wan et al., 2006). Fourth, growing inequality increases calls for more redistribution which deters investment (Alesina & Rodrik, 1994).

Arguments pointing to inequality having a positive effect on economic growth are as follows. Galor and Tsiddon (1997a) present a model in which human capital is determined by home environment externality. When the externality is large, high income inequality may be needed for growth to take off in developing countries. In a second model, Galor and Tsiddon (1997b) develop a model in which technological change leads to a concentration of able workers in technologically advanced industries with the result that high growth is accompanied by high income inequality. Benabou (1996) shows that when human capital of heterogeneous individuals exhibit strong complementarities within localities, more inequality generates higher growth.

The empirical evidence on the effect of inequality on growth for a range of countries and empirical specifications has been mixed (Banerjee & Duflo, 2003). Overall, cross-sectional regressions typically suggest that income inequality has a negative effect on growth, while the findings of panel models with fixed effects suggest income inequality has a positive effect on economic growth (Wan et al., 2006). Banerjee and Duflo (2003) and Forbes (2000) suggest the time horizon is important and that the relationship between inequality and growth is positive in the short and medium-term and negative in the long run. For rural China Ravallion and Chen's (2007) findings point to a negative effect of inequality on growth. In particular they find that periods of most rapid growth were not associated with more rapid increases in inequality, while periods of falling inequality had the highest growth in household income.

Wan et al. (2006) is the only study of which we are aware that systemically examines the effect of income inequality on growth in post-reform China and find that the relationship is nonlinear and negative irrespective of time horizons. Their conclusion (at p.664) is:

Table 7
Head-count ratios for the densities in Fig. 6.

	D_{1990}	M_{2000}	M_{2005}	D_{2005}
$z = 683$	0.129385	0.017173	0.004941	0.026907
$z = 944$	0.309225	0.059482	0.018340	0.062630

“Despite the seemingly positive correlation between growth and inequality in post-reform China, our empirical results unequivocally point to the negative effects of inequality on growth in the short, medium and long runs. The negative effects stem from the strong and negative influence of inequality on physical investment, which consistently outweigh the mostly positive impacts of inequality on human capital”. On the basis of this empirical evidence a stronger redistributive stance from the start of the reforms would not only have had a positive effect on reducing poverty, but might have actually resulted in higher average incomes.

Fig. 6 further explores the effect of rising income inequality on poverty levels. Fig. 6 presents density functions in 2005 prices. OA and OB are the absolute poverty and low-income lines for 2005, being 683 RMB and 944 RMB respectively. The four curves in Fig. 6 are densities of Eq. (14) with different $L(p)$ and μ :

- D_{1990} : $L(p)$ is the estimated Lorenz curve for 1990, μ is average income for 1990 in 2005 prices.
 M_{2000} : $L(p)$ is the estimated Lorenz curve for 1990, μ is average income for 2000 in 2005 prices.
 M_{2005} : $L(p)$ is the estimated Lorenz curve for 1990, μ is average income for 2005 in 2005 prices.
 D_{2005} : $L(p)$ is the estimated Lorenz curve for 2005, μ is average income for 2005 in 2005 prices.

The average income for 1990, 2000 and 2005 are 686 RMB, 2266 RMB and 3365 RMB respectively (SSB, 2007). The average income for 1990 and 2000 in 2005 prices are $\mu_{1990} = 1425$ RMB and $\mu_{2000} = 2475$ RMB respectively. We can thus explicitly give the formulae of the densities. For example, let the Lorenz curve for 1990 and 2005 be respectively $L_{1990}(p)$ and $L_{2005}(p)$. Then the curve D_{1990} is in fact:

$$f(x) = \frac{1}{\mu_{1990} L''_{1990}(p)},$$

where p is obtained by solving $\mu_{1990} L'_{1990}(p) = x$ for a given x . Curve M_{2000} is:

$$f(x) = \frac{1}{\mu_{2000} L''_{1990}(p)},$$

where p is obtained by solving $\mu_{2000} L'_{1990}(p) = x$ for a given x . We estimated the head-count ratios corresponding to these curves which are listed in Table 7.

Since D_{1990} is the density of 1990 in 2005 prices, by the standard of absolute poverty and low-income in 2005, the absolute and low-income head-count ratio for 1990 would be the area OAY and OBX, which are 12.9% and 30.9% respectively as shown in Table 7. Allowing for economic growth, we have D_{2005} in 2005 and the corresponding head-count ratios are in fact 2.69% and 6.26%. This is depicted by the areas OAE and OBD. However, if the total marginal income resulting from economic growth between 1990 and 2005 was distributed proportionally among the rural population (such that the Lorenz curve for 2005 would be the same as that for 1990), the density for 2005 would be M_{2005} . The head-count ratio for M_{2005} , based on the absolute poverty and low-income lines (areas OAF and OBC) are smaller than those for D_{2005} . The areas OAF and OBC are only 0.49% and 1.83% respectively, as shown in Table 7. The poverty indices for 2005 are even larger than those for M_{2000} , which are areas OAI and OBJ. The respective areas are 1.72% and 5.95% as shown in Table 7. This implies that if we could maintain rural income inequality at 1990 levels, then even though the total cake for the whole society is smaller in 2000 than in 2005 (reflecting the fact that average income in 2000 was smaller), the poverty situation in 2000 would have been better compared with the situation in 2005.

5. Conclusion

This paper has introduced a new Lorenz curve model specification and applied it to Chinese rural data over the period 1980 to 2006 to draw conclusions about trends in income inequality. Poverty measures have also been presented and inferences drawn on how inequality has contributed to poverty in recent times. The first main conclusion is that the rural poor have been the big losers from China's economic reforms. Our results suggest that the income share of the rural population at the low end of the income scale has been shrinking. The second conclusion is that income inequality in rural China has increased over time and that income inequality has impeded attempts to reduce poverty. The third conclusion is that in spite of worsening rural poverty and increased rural income inequality, the welfare of the rural population is still improving in terms of the generalized Lorenz dominance criterion.

Our results suggest that there is an urgent need for policies designed to assist the rural poor at the bottom end of the income scale. Other research suggests that policies designed to assist this group need not impinge on overall growth and indeed may be growth enhancing (Wan et al., 2006). What form should such policies take? Our research does not examine the causes of rural poverty. However, previous research suggests that the most effective approach to improving the lot of the rural poor is through a multi-pronged approach. For instance, based on data spanning almost three decades from 1970 to 1997, Fan et al. (2002) argue that the most effective approach is a mixture of investment in human capital, investment in agricultural research and development and investment in rural infrastructure. There are elements of these policies in the State Council's “Number 1 Document” issued in 2006 (Chow, 2006).

Such policies could be coupled with programs targeted to areas experiencing high levels of rural poverty. The central and provincial governments have invested in “poor area development programs”, many of which have delivered reasonable rates of return (Ravallion & Jalan, 1999). At a more general level, several studies have emphasized that farmers' rights have been violated by illegal activities of local government officials including confiscation of land without adequate compensation (Chen, 2006; Chow, 2006). This point relates not to poverty in the narrow sense of low income *per se*, but is concerned with the economic welfare of

low income families when their property rights are violated. The National People's Congress passed the Rural Land Contract Law in 2003 which outlaws illegal seizure, but enforcement has been problematic. Wen Jiabao stated in 2006 that enforcement must be improved to avoid illegal seizures as part of a strategy to reduce rural protests (Chow, 2006).

In terms of future research, on the methodological front, one of the challenges of using grouped data to study income inequality in China is to develop more efficient Lorenz models to approximate the underlying Lorenz curve. This paper shows that the basic Lorenz model of $\tilde{L}(p) = p^\alpha L(p)^\eta$ and the concept of the ordered family of Lorenz curves developed by Sarabia et al. (1999), are important ideas. However, we also show that the condition imposed by Sarabia et al. (1999) for the models to be Lorenz curves proves to be restrictive. Theorem 3 in this paper ensures that we can construct new Lorenz models from almost any parametric function $L(p)$ so long as it satisfies the condition of the Lorenz curve. Moreover, we can find even better performing Lorenz models if the condition $L'''(p) \geq 0$ is satisfied.

Further research could use theorem 3 developed in this paper to find other $L(p)$, in order to create more efficient Lorenz models with the basic form $\tilde{L}(p)$. Future research could build on the methodological contribution here to find other models which are efficient and, as such, have larger admissible ranges of η . One of the limitations of our results is that we have said nothing about the huge urban-rural income gap and spatial income differences in China. Future research could use more efficient Lorenz models to examine the rural-urban divide in China or do a comparative study of income distribution between China and other countries.

Appendix A. Proof of Theorem 3

Consider the first statement. We only have to prove $\tilde{L}'(p) \geq 0$ and $\tilde{L}''(p) \geq 0$. Note

$$\tilde{L}'(p) = \alpha p^{\alpha-1} L(p)^\eta + \eta p^\alpha L(p)^{\eta-1} L'(p), \quad (\text{A1.1})$$

$$\frac{\tilde{L}''(p)}{p^{\alpha-2} L(p)^{\eta-2}} = \alpha(\alpha-1)L(p)^2 + \alpha\eta p L(p)L'(p) + \eta p^2 [(\eta-1)L'(p)^2 + L(p)L''(p)] + \alpha\eta p L(p)L'(p).$$

Obviously, $\tilde{L}(p)$ is a Lorenz curve when $\alpha \geq 1$. Assume $\alpha \in [0, 1]$. It is sufficient for $\tilde{L}(p)$ to be a Lorenz curve if the sum of the first two terms on the right side of Eq. (A1.1) is non-negative or

$$(\alpha-1)L(p) + \eta p L'(p) \geq 0. \quad (\text{A1.2})$$

This inequality clearly holds since the function on the left side of the inequality is equal to zero at $p=0$ and is increasing on $[0, 1]$ from the derivative of the function.

Next consider the second statement. Note under the present assumptions that the inequality in Eq. (A1.2) still holds, so that we only need to prove that the sum of the last two terms on the right side of Eq. (A1.1) is non-negative. Let the sum of the last two terms on the right hand side of Eq. (A1.1) be denoted as $g(p)$ after being divided by ηp . We only need to prove that:

$$g(p) = p [(\eta-1)L'(p)^2 + L(p)L''(p)] + \alpha L(p)L'(p) \geq 0 \text{ for all } p \in [0, 1] \quad (\text{A1.3})$$

holds for any $\eta \in [1/2, 1]$ and $\alpha \geq 0$ satisfying $\alpha + \eta \geq 1$. This inequality is true given the assumptions, because $g(p)$ is increasing on $[0, 1]$ from its derivative and satisfies $g(0) = 0$. \square

Appendix B. Proof of Theorem 4

It is straightforward to verify that

$$H'_0(p) = [\beta + \gamma(1-p)](1-p)^{\beta-1} e^{-\gamma p}, \quad H''_0(p) = \{\beta - [\beta + \gamma(1-p)]^2\}(1-p)^{\beta-2} e^{-\gamma p}.$$

Thus, H_0 is a Lorenz curve for all (β, γ) satisfying $\beta > 0$ and $0 \leq \beta + \gamma \leq \sqrt{\beta}$, and the first part of theorem 3 implies $H_3(p)$ is a Lorenz curve under condition (10).

Note

$$\frac{H'''_0(p)}{(1-p)^{\beta-3} e^{-\gamma p}} = 2\gamma[\beta + \gamma(1-p)](1-p) + \{\beta - [\beta + \gamma(1-p)]^2\}\{2 - [\beta + \gamma(1-p)]\}.$$

Denote the function on the right side of this equation as $f(p)$. We can claim that $f(p) \geq 0$ and, consequently, $H'''_0(p) \geq 0$ for all $p \in [0, 1]$ if (β, γ) satisfies:

$$\beta \in (0, 1], \quad 0 \leq \beta + \gamma \leq \sqrt{\beta}. \quad (\text{A2.1})$$

The statement is true if $\gamma \geq 0$. Consider the case if $\gamma < 0$. The condition in (A2.1) implies $f'(p) \leq 0$ so that $f(p)$ is a decreasing function on $[0, 1]$. But $f(1) = \beta(1-\beta)(2-\beta) \geq 0$ for any $\beta \in (0, 1]$. Thus, $f(p) \geq 0$ does hold for all $p \in [0, 1]$ if condition (A2.1) is true. Drawing on the second part of theorem 3, $H_3(p)$ is a Lorenz curve if the condition specified in Eq. (11) is satisfied. \square

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