A Decision-Based Model and Algorithm for Maneuvering Target Tracking

JIAHONG CHEN
ZHONGHUA ZHANG
ZHENDONG XI
YONGXING MAO
China Satellite Maritime Tracking and Control Department,
Jiangyin, Jiangsu, 214 431
stone_cjh@sina.com; zhangzh_smtc@sina.com; dong3691@hotmail.com; myx8282@Tom.com

Abstract: The decision-based models for maneuvering target tracking were studied in this paper. Focusing on the problem of dissatisfied with single model tracking and the optimal model-set is difficult to design of the multiple-model (MM) algorithm, we modified the "current" model, and proposed an adaptive single model (ASM) to track angular motion. The united velocity-acceleration estimation-based and direct measurement-based decision algorithms for model-switching were provided and discussed. At the same time, we discussed the question of the thresholds setup, and gave a significance test method based on hypothesis testing theory. Simulation results show that the tracking performance of this new adaptive single model is stationary, and is improved much more than the "current" model, also better than the interacting multiple-model (IMM) for strong maneuvering target tracking. Furthermore the computational load of the new model can be less than that of IMM.

Key-Words: adaptive single model, multiple-model, maneuvering target tracking, "current" model, decision-based algorithm, model-switching

1 Introduction

The key to successful target tracking lies in the effective extraction of useful information about the target's state from observations. A good model of the target will certainly facilitate this information extraction to a great extent. [1-6] gave a comprehensive and up-to-date survey of the techniques for tracking maneuvering target, especially of the tracking models. Interrelationships among models and insight to the pros and cons of models were provided by [1].

In the history of the development of maneuvering target tracking (MTT) techniques, single model based adaptive Kalman filter free of decision came into existence first. As one single model can only match one motion mode best, and the performance of tracking time-varying or strong maneuvering target is dissatisfied, so the decision-based algorithm was proposed, by [4], which belonged to the switching-model approach in a broad sense. The followed approaches of multiple-model (MM) algorithms have become quite popular for MTT, among which the interacting multiple-model (IMM) is more preferable [5-10].

The universal maneuvering target tracking techniques based on single model are adaptive Kalman filtering algorithms [11, 12, 13], which faced the difficulties to find a suitable bank of parameters to match the system motion mode precisely. The algorithms, which use statistical methods with multi-banks of parameters to describe the motion mode approximately, were called MM algorithms. To improve the tracking performance of MM will face to complex model-set designing and burdened calculating. It was shown that the system is optimal only when there is one model in the model-set and the model also must match the motion mode completely by [8]. Here we will study the decision-based algorithms and try to avoid the complex designing of the model-set and to find a more suitable single model for MTT.

As surveyed by [4], for MTT, there are three main classes of decision-based techniques: equivalent noise, input detection and estimation, and switching model, which are based on decisions regarding to target maneuver. In a broad sense, algorithms in the equivalent noise or input detection and estimation approaches also belong to the switching model approach since following different decisions these algorithms taking different actions, which may be construed as filtering based on different models. The fundamental problem is the detection of maneuver onset and termination. Two popular choices for the detection term are the measurement residual $\tilde{z}$ and the input estimate $\boldsymbol{u}$, others are based of them or of there combined functions.

In the open literature available to us, one decision-based algorithm usually uses only one detection term, which can not be more adaptive to the time-varying maneuver generally. Moreover there are only two single models in essence which are switched each other, e.g., between CV (constant-velocity) and CA (constant-acceleration) [14] or between CV and Singer (Singer Acceleration) Models [15], etc. The methods can generally be called as estimation-based methods where $\boldsymbol{u}$ and $\tilde{z}$ or v (velocity) are all estimation-based. The precision of estimation is relative to the system noise (process noise and measurement noise) and model reliability. Generally the infection of noise can be decreased by model filtering. We can also take the direct measurement as detection term which will be simple. Furthermore the validity of model is irrelative to the decision, whereas the noise infection will be increased. In this paper we intend to propose a united velocity-acceleration estimation-based decision algorithm (EBDA) and a direct measurement-based decision algorithm (MBDA) with a new single united maneuvering and non-maneuvering model.

The rest of the paper is organized as follows. Section 2 presents a new adaptive model. The united velocity-acceleration estimation-based decision algorithm and thresholds setup are presented in Section 3 and 4. Section 5 gives the measurement-based decision algorithm with the new model. Comparison of the new algorithms and IMM for simulated different trajectories is given in section 6. Concluding remarks are provided in Section 7.

2 The New Adaptive Single Model

2.1 The Singer Model and "Current" Model

The CV model is more proper for non-maneuvering target, while the CA model is more suitable for the motion whose acceleration derivative (i.e., jerk) $\dot{a}(t)$ is an independent process (white noise) $w(t)$: $\dot{a}(t) = w(t)$.

The Singer model assumes that the target acceleration $a(t)$ is a zero-mean first-order stationary Markov process with autocorrelation $R_a(\tau) = \mathrm{E}[a(t+\tau)a(t)] = \sigma^2 e^{-\alpha|\tau|}$. Such a process $a(t)$ is the state process of a linear time-invariant system:

$$\dot{a}(t) = -\alpha a(t) + w(t) \tag{1}$$

where $a = \ddot{x}$, is the acceleration; $\sigma^2 = \mathrm{E}[a(t)^2]$ is the "instantaneous variance" of the acceleration; $\alpha = 1/\tau$ is the reciprocal of the maneuver time constant τ and thus depends on how long the maneuver lasts. For example for an aircraft, $\tau \approx 60s$ for a lazy turn and $\tau \approx 10--20s$ for an evasive maneuver, for the missile τ maybe a little longer [1]; $w(t)$ is zero-mean white noise with constant power spectral density $S_w = 2\alpha\sigma^2$. As the maneuver time constant τ increases or decreases the Singer model corresponds to a motion in between of (nearly) CA and (nearly) CV. It should thus be clear that the Singer model has wider coverage than CV and CA models [1, 6].

Another acceleration model, called the "current" model by its authors, proposed by [16], is in essence a Singer model with an adaptive mean of acceleration that is mean-adaptive acceleration (MAA) model:

$$\dot{a}(t) = -\alpha a(t) + \alpha\bar{a}(t) + w(t) \tag{2}$$

With state vector $\boldsymbol{x} = \begin{bmatrix} x & \dot{x} & \ddot{x} \end{bmatrix}^{\mathrm{T}}$, where the superscript T denotes the transpose of matrix, the state-space representation of this model is:

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\alpha \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix} \bar{a}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \boldsymbol{w}(t) \tag{3}$$

excluding the time instants at which samples are taken since $\bar{a}(t)$ is assumed piecewise constant. The discrete-time equivalent is:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \begin{bmatrix} 1 & T & (-1+\alpha T + e^{-\alpha T})/\alpha^2 \\ 0 & 1 & (1-e^{-\alpha T})/\alpha \\ 0 & 0 & e^{-\alpha T} \end{bmatrix} \boldsymbol{x}_k + \\ &\quad \begin{bmatrix} (T^2/2) - (-1+\alpha T + e^{-\alpha T})/\alpha^2 \\ T - (1-e^{-\alpha T})/\alpha \\ 1-e^{-\alpha T} \end{bmatrix} \bar{a}_k + \boldsymbol{w}_k \\ &= F_\alpha \boldsymbol{x}_k + B_\alpha \bar{a}_k + \boldsymbol{w}_k \end{aligned} \tag{4}$$

where T is the sampling interval. As $\alpha T << 1$, the covariance of the noise w_k is given by:

$$Q_k = 2\alpha\sigma^2 \begin{bmatrix} T^5/20 & T^4/8 & T^3/6 \\ T^4/8 & T^3/3 & T^2/2 \\ T^3/6 & T^2/2 & T \end{bmatrix} \tag{5}$$

A key underlying assumption of the "current" model given by [16] (but not so stated explicitly) is that $\bar{a}_{k+1} = \hat{a}_{k|k}$, that is to use the last estimation as the current mean. This is questionable and can be actually avoided [1, 6]. Getting the expectation of the last item of (4), we have:

$$\bar{a}_{k+1} = E[a_{k+1} \mid z_k] = e^{-\alpha T} E[a_k \mid z_k] + (1 - e^{-\alpha T})\bar{a}_k$$
$$= e^{-\alpha T}\hat{a}_{k|k} + (1 - e^{-\alpha T})\bar{a}_k = \hat{a}_{k+1|k} \quad (6)$$

where z_k is the measurement vector.

2.2 Modification to the "Current" Model

Application of the "current" model can be found in the literatures. A weighted "current" model adaptive Kalman filtering algorithm is suggested by [17], while an improved adaptive filtering algorithm of noise variances self-adaptation was given by [18] based on the "current" statistical model. Here we take into account the matching problem of the model. The process of target tracking is the servo system of radar driving antenna to point to the target with time-varying angular velocity and acceleration controlling instruction. By the sense of physics, it is viable to use mean-adaptive acceleration model to match the relative angular motion of the target. Actually, the angular velocity and acceleration are often time-varying for radar target tracking, especially for rectilineal motion with constant velocity the angular motion is not constant.

As $\tau \geq 10s$ and the sampling interval T is small ($\leq 0.5s$, it's possible), expand $e^{-\alpha T}$ of the current model (4) with Taylor series:

$$e^{-\alpha T} = 1 - \alpha T + \frac{1}{2!}(\alpha T)^2 - \frac{1}{3!}(\alpha T)^3 + \cdots\cdots \quad (7)$$

Substituting (7) into (4), ignoring the high order item, we can obtain nearly "current" model:

$$\boldsymbol{x}_{k+1} = \begin{bmatrix} 1 & T & T^2/2 - \alpha T^3/6 \\ 0 & 1 & T - \alpha T^2/2 \\ 0 & 0 & 1 - \alpha T \end{bmatrix} \boldsymbol{x}_k + \begin{bmatrix} \alpha T^3/6 \\ \alpha T^2/2 \\ \alpha T \end{bmatrix} \bar{a}_k + \boldsymbol{w}_k \quad (8)$$

It's not difficult to find that the result of (6) is also correct for (8).

As the new model should be universal and adaptive, we modified the model of (8) by importing the adaptive factor λ:

$$\boldsymbol{x}_{k+1} = \begin{bmatrix} 1 & T & \lambda(T^2/2 - \alpha T^3/6) \\ 0 & 1 & \lambda(T - \alpha T^2/2) \\ 0 & 0 & \lambda(1 - \alpha T) \end{bmatrix} \boldsymbol{x}_k + \begin{bmatrix} \alpha T^3/6 \\ \alpha T^2/2 \\ \alpha T \end{bmatrix} \bar{a}_k + \boldsymbol{w}_k \quad (9)$$

Denotes (9) as:

$$\boldsymbol{x}_{k+1} = F_k \boldsymbol{x}_k + B_k \bar{a}_k + \boldsymbol{w}_k \quad (10)$$

where λ only can be 0 or 1, α is also a fixed value (in this paper we choose 0.1) or zero. Analyzing the model of (9), we can see that:

1) When α and $\bar{a}_k$ are not equal to zero, and $\lambda = 1$, the system is nearly equivalent to MAA model;

2) When $\alpha \neq 0$ and $\lambda = 1$ but $\bar{a}_k = 0$, the system is nearly equivalent to Singer model;

3) When $\alpha = 0$ and $\lambda = 1$, the system is nearly equivalent to CA model but for random noise item;

4) When $\alpha = 0$ and $\lambda = 0$, the system is nearly equivalent to CV model but for random noise item.

We call the universal model of (9) as improved adaptive single model (ASM). It's not difficult to find that if the parameter α and λ varying in real time, this new model can match constant-velocity, constant-acceleration and time-varying acceleration motion, so it has strong adaptability.

3 United Estimation-Based Decision Algorithm

3.1 Adaptive Rules

To match the target motion exactly, ASM should be switched in real time between MAA, CA and CV according to the measurements and states estimation. We will design the adaptive rules with united velocity-acceleration estimations by distinguishing three kinds of situations.

The mean (average) and modified standard deviation (MSD) of acceleration and velocity in continuous m steps are:

$$\mu_{a,m} = \frac{1}{m}\sum_{i=1}^{m} \hat{a}_{k-i|k-i} \quad (11)$$

$$\sigma_{a,m} = \Big(\frac{1}{m-1}\sum_{i=1}^{m} (\hat{a}_{k-i|k-i} - \mu_{a,m})^2\Big)^{1/2} \quad (12)$$

$$\mu_{v,m} = \frac{1}{m}\sum_{i=1}^{m} \hat{v}_{k-i|k-i} \quad (13)$$

$$\sigma_{v,m} = \Big(\frac{1}{m-1}\sum_{i=1}^{m} (\hat{v}_{k-i|k-i} - \mu_{v,m})^2\Big)^{1/2} \quad (14)$$

where k denotes current step of filtering and estimating. Here m can not be too large because of maneuver will take place at any time, so we use the modified standard deviation instead of the standard deviation by statistical theory. The adaptive rules for model-switching are followed as:

1) If the system is under MAA model currently ($\lambda = 1$, $\alpha \neq 0$)

(a). Meeting the condition $\sigma_{a,m} < \delta_1$, i.e., when the MSD of acceleration of the final m steps is small enough, the model should be switched to CA model, which means $\lambda = 1$, $\alpha = 0$.

(b). Meeting the condition $\mu_{a,m} < \delta_2$ and $\sigma_{v,m} < \delta_3$, i.e., when the MSD of velocity of the final m steps is small enough and the mean of acceleration is near to zero, the model should be switched to CV model, which means $\lambda = 0$, $\alpha = 0$.

2) If the system is under CA model currently ($\lambda = 1$, $\alpha = 0$)

(a). The (b) of (1) is still applicable.

(b). Meeting the condition $\sigma_{a,m} > \delta_4$, i.e., when the MSD of acceleration of the final m steps is large enough, the model should be switched to MAA model, which means $\lambda = 1$, $\alpha \neq 0$.

3) If the system is under CV model currently ($\lambda = 0$, $\alpha = 0$)

Meeting the condition $\sigma_{v,m} > \delta_5$, i.e., when the MSD of velocity of the final m steps is large enough, the model should be switched to CA model, which means $\lambda = 1$, $\alpha = 0$.

If the conditions are not met, the model should be hold.

3.2 State Estimation in Spherical Coordinate-System

Many studies of target tracking are focused on describing the motion mode in inertial Cartesian coordinate-system (CS). But radar system can only provide the bearing (or azimuth) b and elevation e measurements, and possibly some radar can also provide range r or range rate (Doppler) $\dot{r}$ (in spherical CS). To study target motion in the Cartesian CS is the direct method, but the complex nonlinear model must be introduced because of no x, y and z measurements in Cartesian CS. All kinds of approximation and transformation will bring on bias of angular controlling instruction. We consider that studying independently of radar tracking in spherical CS from state estimation in Cartesian CS is doable, because in spite of any way the radar and target moving, the essence of tracking is to ascertain the relative bearing and elevation angular motion of the target promptly and exactly. An angular error tracking filter and model was researched by Ekstrand B. in [19], where only the nearly constant-velocity (CV) model was used to estimate the inertial angular rate of the line-of-sight (LOS), whereas no optimal angle estimation directly. Here we will estimate the angle directly.

Because the range r, bearing b and elevation e in spherical coordinate-system of the radar system can be considered as independently, and the state and measurement equations of the three channels are the same, so we can filter and estimate the three channels independently with the same algorithm. The state equation of the system is just like (9), whereas the measurement equation is:

$$\begin{aligned} \boldsymbol{z}_k &= H\boldsymbol{x}_k + \boldsymbol{v}_k \\ &= [1 \quad 0 \quad 0]\boldsymbol{x}_k + \boldsymbol{v}_k \end{aligned} \tag{15}$$

where $\boldsymbol{z}_k$ denotes the measurement and $\boldsymbol{v}_k$ is the measurement noise, the covariance of $\boldsymbol{v}_k$ is R_k. We can see that the model is linear, despite it is time-varying, so the optimal Kalman filter can be used here.

To take the bearing b channel for example, denotes $\boldsymbol{x} = \begin{bmatrix} b & \dot{b} & \ddot{b} \end{bmatrix}^{\mathrm{T}}$, than the EBDA for state estimation mainly consists of two steps: value F_k, B_k by the adaptive rules and estimate the state with Kalman filter. The complete steps for the state estimation algorithm are concluded as in Table 1.

We proposed the new model based on angular motion of radar target tracking, which can match the uniform motion and variable motion together. It is suitable for range channel at the same time too, which is just the exhibition of the strong adaptability of the new model algorithm.

Table 1 The EBDA of ASM for State Estimation

1. value F_k and B_k by the adaptive rules	
2. model-conditioned filtering (if switching filter from $k-m$)	
Predicted state:	$\hat{\boldsymbol{x}}_{k+1\|k} = F_k \hat{\boldsymbol{x}}_{k\|k} + B_k \bar{a}_k$
Predicted measurement:	$\hat{\boldsymbol{z}}_{k+1\|k} = H\hat{\boldsymbol{x}}_{k+1\|k}$
Predicted covariance:	$P_{k+1\|k} = F_k P_{k\|k} F_k^{\mathrm{T}} + Q_k$
Measurement residual:	$\tilde{\boldsymbol{z}}_{k+1} = \boldsymbol{z}_{k+1} - \hat{\boldsymbol{z}}_{k+1\|k}$
Residual covariance:	$S_{k+1} = HP_{k+1\|k} H_k^{\mathrm{T}} + R_{k+1}$
Filter gain:	$K_{k+1} = P_{k+1\|k} H^{\mathrm{T}} S_{k+1}^{-1}$
Updated state:	$\hat{\boldsymbol{x}}_{k+1\|k+1} = \hat{\boldsymbol{x}}_{k+1\|k} + K_{k+1}\tilde{\boldsymbol{z}}_{k+1}$
Updated covariance:	$P_{k+1\|k+1} = P_{k+1\|k} - K_{k+1} H P_{k+1\|k}$

4 Thresholds Setup

The key to successful performance improving of ASM tracking lies in the exact setup of the thresholds δ_1, δ_2, δ_3, δ_4 and δ_5. Besides experiment and experience, we will analyze in view of probability and statistics theory.

4.1 Thresholds of Variable Motion

Take the statistic of acceleration for example. If the system moves with constant acceleration, we can assume the acceleration has Gaussian distribution $N(\mu_a, \sigma_a^2)$. The first moment (mean) μ_a varies by the setup, whereas the second moment (covariance) σ_a^2 is relatively fixed, and depends on the performance of the system, such as sampling interval, measurement precision, etc.. We also can say that σ_a^2 is determined by the system, and can be obtained approximately by experiment sampling. If $\sigma_{a,m} \le \sigma_a$ obviously under MAA model, we can consider the acceleration is nearly constant reasonably; on the other hand, if $\sigma_{a,m} > \sigma_a$ obviously under CA model, modify the model to MAA is also believable.

Definition 1: assume $F(y)$ is the distillation function of random variable Y, p is a real number, $0 \le p \le 1$, if:

$$P\{Y \le y_p\} = F(y_p) = p \tag{16}$$

where $P\{\bullet\}$ stands for probability. We call y_p the p percentile of the distribution.

To ascertain δ_1 and δ_4 is a problem of hypothesis testing.

1) Inferior (lower) limit δ_1

We establish the original hypothesis H_0: $\sigma_{a,m}^2 \ge \sigma_a^2$ against the alternative hypothesis H_1: $\sigma_{a,m}^2 < \sigma_a^2$.

Definition 2: Type Ⅰ error is the error made when rejecting H_0 while H_0 is true. Type Ⅱ error is the error made when accepting H_0 while H_1 is true.

To control the risk of mistaking $\sigma_{a,m}^2 < \sigma_a^2$ when $\sigma_{a,m}^2 \ge \sigma_a^2$ actually, i.e., the Type Ⅰ error is less than β (significance level).

According to the Gaussian distribution of a, the random variable $\frac{1}{\sigma_a^2}\sum_{i=1}^{m}(\hat{a}_{k-i|k-i} - \mu_{a,m})^2$ is chi-square distributed ($\chi^2_{(m-1)}$), so the rejection region W_1 (rejecting H_0 and accepting H_1) is:

$$\left\{ (\hat{a}_{k-1|k-1},\ldots\ldots,\hat{a}_{k-m|k-m}) : \frac{1}{\sigma_a^2}\sum_{i=1}^{m}(\hat{a}_{k-i|k-i} - \mu_{a,m})^2 = \frac{(m-1)\sigma_{a,m}^2}{\sigma_a^2} \le \chi^2_{\beta(m-1)} \right\} \tag{17}$$

$\chi^2_{\beta(m-1)}$ is the β percentile of $\chi^2_{(m-1)}$ distribution. If β is small, rejecting H_0 and accepting H_1 is credible by "event with small probability should not take place in one experiment".

Here we take $\beta = 0.1$ and $m = 5$, query the χ^2 distribution table, $\chi^2_{0.1(4)} = 1.064$, so when $\sigma_{a,m} \le 0.516\sigma_a$, (17) comes into existence. That is to say we can set $\delta_1 = \sigma_a / 2$.

2) Superior (upper) limit δ_4

Here we should control the risk of mistaking $\sigma_{a,m}^2 > \sigma_a^2$ when $\sigma_{a,m}^2 \le \sigma_a^2$ actually, so we establish the original hypothesis H_0: $\sigma_{a,m}^2 \le \sigma_a^2$ against the alternative hypothesis H_1: $\sigma_{a,m}^2 > \sigma_a^2$. The rejection region W_1 of the random variable $\frac{1}{\sigma_a^2}\sum_{i=1}^{m}(\hat{a}_{k-i|k-i} - \mu_{a,m})^2$ is:

$$\left\{ (\hat{a}_{k-1|k-1},\ldots\ldots,\hat{a}_{k-m|k-m}) : \frac{1}{\sigma_a^2}\sum_{i=1}^{m}(\hat{a}_{k-i|k-i} - \mu_{a,m})^2 = \frac{(m-1)\sigma_{a,m}^2}{\sigma_a^2} > \chi^2_{1-\beta(m-1)} \right\} \tag{18}$$

If $\beta = 0.1$, then $\chi^2_{0.9(4)} = 7.779$, so when $\sigma_{a,m} > 1.395\sigma_a$, (18) comes into existence. That is to say we can set $\delta_4 = 1.395\sigma_a$. If we set $\delta_4 = 2\sigma_a$, then $\beta < 0.005$, so the reliability of H_1: $\sigma_{a,m}^2 > \sigma_a^2$ is very high (> 0.995).

In brief, we can set δ_1 near to $\sigma_a / 2$ and δ_4 near to $2\sigma_a$ from the above analysis. In fact the setup of δ_1 and δ_4 should also accord with the experiment results. Two questions must be paid attention to. The first is that $\sigma_a / 2$ and $2\sigma_a$ are only reference values because m can not be too large by the real-time requirement. The deterministic value must be experienced typical experiments. Second, $\sigma_{a,m}$ is relative to the sampling interval, also same with σ_a, so when T is varied the thresholds should be varied accordingly.

4.2 Thresholds of Uniform Motion

The methods of setup of δ_3 and δ_5 resemble to the above δ_1 and δ_4. Assuming the system is under CV model, and the distribution of velocity is $N(\mu_v, \sigma_v^2)$, so δ_3 can be near to $\sigma_v / 2$, and δ_5 can be near to $2\sigma_v$.

The matching performance of CV model is inferior to that of CA and MAA model except that the system is actually moving with constant velocity. Switching the CA and MAA model to CV model should be cautious, so we appended the constraint of acceleration mean: $\mu_{a,m} < \delta_2$. The physical meaning of this condition is obvious.

If the acceleration noise is $N(0,\sigma_a^2)$, we establish the original hypothesis H$_0$: $\mu_{a,m} = 0$ against the alternative hypothesis H$_1$: $\mu_{a,m} \neq 0$.

The random variable $\sqrt{m}\dfrac{\mu_{a,m}-0}{\sigma_a}$ is standard Gaussian distribution $N(0,1)$, so the confidence interval W$_0$ should meet:

$$\sqrt{m}\frac{|\mu_{a,m}|}{\sigma_a} < u_{1-\beta/2} \quad (19)$$

where $u_{1-\beta/2}$ is the percentile of Gaussian distribution, $\beta = 0.1$ as the above tests, so we have:

$$|\mu_{a,m}| < \frac{u_{1-\beta/2}}{\sqrt{m}}\sigma_a = \frac{u_{0.95}}{\sqrt{5}}\sigma_a = \frac{1.645}{\sqrt{5}}\sigma_a = \frac{\sigma_a}{1.36} \quad (20)$$

δ_2 is near to $\sigma_a/1.36$. Different from the above tests, accepting H$_0$ should be credible here. Here we should also control the Type Ⅱ error, i.e., control the risk of mistaking $\mu_{a,m} = 0$ when $\mu_{a,m} \neq 0$ actually. As m can not be too large, according to the hypothesis testing theory we can increase β. In order to be more credible of H$_0$: $\mu_{a,m} = 0$, δ_2 can be set between $(\frac{1}{2} \sim \frac{1}{10})\sigma_a$ conservatively by experiment.

By the way, the short sampling interval and suitably large m can improve the stability of statistical data, and is of advantage to match motion mode in real-time.

5 Direct Measurement-Based Decision Algorithm and Discussion

5.1 Adaptive Rules

The first-order difference of position ($z_{1,k}$) is relative to the target velocity. The second-order difference of position ($z_{2,k}$) is relative to the target acceleration, and the third-order difference of position ($z_{3,k}$) is relative to the target acceleration derivative. Denote:

$$z_{1,k} = z_{k+1} - z_k \quad (21)$$

$$z_{2,k} = z_{1,k+1} - z_{1,k} \quad (22)$$

$$z_{3,k} = z_{2,k+1} - z_{2,k} \quad (23)$$

Regarding of the randomicity of measurement, the average (mean) of the n th-order difference in m steps is used here (denote as $\bar{z}_{1,k}$, $\bar{z}_{2,k}$ and $\bar{z}_{3,k}$). We give the thresholds of velocity $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$, and of acceleration $\boldsymbol{\eta}_3$ and $\boldsymbol{\eta}_4$.

1) If the system is under MAA model currently ($\lambda = 1$, $\alpha \neq 0$)

(a). Meeting the condition $\bar{z}_{3,k} < \boldsymbol{\eta}_3$ and $\bar{z}_{2,k} < \boldsymbol{\eta}_1$, i.e., when the acceleration and its derivative of the final m steps are small enough simultaneously, the model should be switched to CV model, which means $\lambda = 0$, $\alpha = 0$.

(b). Meeting the only condition $\bar{z}_{3,k} < \boldsymbol{\eta}_3$, i.e., when the acceleration derivative of the final m steps is small enough, the model should be switched to CA model, which means $\lambda = 1$, $\alpha = 0$.

2) If the system is under CA model currently ($\lambda = 1$, $\alpha = 0$)

(a). Meeting the condition $\bar{z}_{3,k} > \boldsymbol{\eta}_4$, i.e., when the acceleration derivative of the final m steps is large enough, the model should be switched to MAA model, which means $\lambda = 1$, $\alpha \neq 0$.

(b). Meeting the condition $\bar{z}_{3,k} < \boldsymbol{\eta}_3$ and $\bar{z}_{2,k} < \boldsymbol{\eta}_1$, i.e., when the acceleration and its derivative of the final m steps are small enough simultaneously, the model should be switched to CV model, which means $\lambda = 0$, $\alpha = 0$.

3) If the system is under CV model currently ($\lambda = 0$, $\alpha = 0$)

(a). Meeting the condition $\bar{z}_{2,k} > \boldsymbol{\eta}_2$ and $\bar{z}_{3,k} > \boldsymbol{\eta}_4$, i.e., when the acceleration and its derivative of the final m steps are large enough simultaneously, the model should be switched to MAA model, which means $\lambda = 1$, $\alpha \neq 0$.

(b). Meeting the only condition $\bar{z}_{2,k} > \boldsymbol{\eta}_2$, i.e., when the acceleration of the final m steps is large enough, the model should be switched to CA model, which means $\lambda = 1$, $\alpha = 0$.

The estimation steps and thresholds setup of this MBDA are similar to that of EBDA in Table 1.

5.2 Discussion

The new algorithms of ASM can switch to the better matching model based on the analysis of the state estimation or the direct measurements. If the real target trajectory is taken as an unknown complex curve, than the new ASM will fit the mode comprehensively with first degree, quadratic and cubic curves. It is more adaptive than the traditional single models or switching-model approaches only with first degree and quadratic curves. If the matching model of ASM is appropriate, than the tracking performance will be improved especially for the strong maneuvering targets. The simulation results have shown this well.

We give a decision-based switching-model approach for MTT, which is different from the traditional switching-model approaches. First, the nearly n th-order ($n=1,2,3$) models were used to fit the motion mode, the fitting performance of the new algorithm should be better. Second, the switched models have a uniform expression, model-switching are realized by revaluing the parameters. Third, the union of velocity, acceleration and jerk is taken as the detection item rather than that only one of them is used in the traditional switching-model approaches. Forth, the physical sense of the detection term and the thresholds are obvious and understandable. One important assumption is that if the system moves with constant acceleration (or constant velocity), the acceleration (or velocity) has Gaussian distribution.

In the new algorithms, if the mode variation was detected at time k , the model should be switched from $k-m$. When mT is small enough (< τ), switching from k or $k-m$ has no obvious difference. Switching from k can lighten the computational load. On the other hand if τ is smaller, the maneuver acceleration can be considered as noise.

For EBDA, considering the statistical reliability of the detection terms, the step m can not be too small. With the acceleration to be estimated, $m>3$. In view of real-time computation, m can not be too large. Generally when $m<=10$, the computational load is nearly approximate to the IMM algorithm. For the computational system with high performance, if T is very small, than m can be larger.

For MBDA, the third-order difference should be calculated, then to compute the average. The covariance of the n th-order difference is in (direct) proportion to 2^n (it can be proved about $2^n\sigma_z^2$, σ_z^2 is the variance of measurement z). m should be large enough to reduce the infection of noise, $m>5$ generally. Same as the EBDA m also can not be too large ($mT<\tau$). The complexity of the two algorithms corresponds to each other. From the simulation results we can see that the tracking performance of MBDA+ASM is also satisfied, which shows the strong adaptability of ASM.

6 Simulation

In this section, we compared the performance of ASM with EBDA to the classic MAA and IMM algorithms by simulation. MBDA based of ASM is also simulated.

6.1 Design of Simulation

With the origin O at the radar center of the three axis (range, bearing, elevation), we assume the target moves in a plane with fixed $z=10000$ m. The state vector is $[x\ \dot{x}\ y\ \dot{y}]^{\mathrm{T}}$ in Cartesian CS, whereas $[r\ \dot{r}\ \ddot{r}]^{\mathrm{T}}$, $[b\ \dot{b}\ \ddot{b}]^{\mathrm{T}}$ and $[e\ \dot{e}\ \ddot{e}]^{\mathrm{T}}$ in spherical CS. Simulation time is 0~100s.

The model-set of IMM has one CV model and two CT (Constant-Turn) models. There is always one model in the model-set which is matched to the system motion mode completely. In this point, the result of IMM algorithm in this simulation is preferable to the adaptive IMM algorithm. The transition probability matrix is:

$$p_{ij}=\begin{bmatrix}0.8 & 0.1 & 0.1\\ 0.1 & 0.8 & 0.1\\ 0.1 & 0.1 & 0.8\end{bmatrix} \tag{24}$$

and the model initialized probability is:

$$\mu_0=\begin{bmatrix}0.5 & 0.3 & 0.2\end{bmatrix}^{\mathrm{T}} \tag{25}$$

The design of MAA and ASM simulation is a little more complex. To compare the performance exactly, we use the measurements which were created in Cartesian CS containing process noise and measurement noise, after Cartesian-to-Spherical transformation, and obtain the measurements in Spherical CS. When finished the simulation, we transform the estimating result in Spherical CS to Cartesian CS, and then compare to the true trajectory. In this simulation $T=0.05$, $m=5$ for EBDA, $m=10$ for MBDA. Pay attention that the filtering noise matrix in Spherical CS is relative to the noise in Cartesian CS but not equal to. The performance of MAA and ASM simulation is very conservative and will be better in practice engineering in this point.

We give the index of root mean square (RMS), normalized position error (NPE) and computational load (CL). The CL of the algorithm is the mean value which is evaluated in terms of calculating time with respect to Monte Carlo runs. The formulas of calculating RMS and NPE are:

$$RMS(x,k)=\left(\frac{1}{M}\sum_{j=1}^{M}[\tilde{x}_k(j)\tilde{x}_k(j)]\right)^{1/2} \tag{26}$$

$$\overline{RMS}(x)=\frac{1}{N}\sum_{k=1}^{N}RMS(x,k) \tag{27}$$

$$NPE(j) = (\sum_{k=1}^{N}[\tilde{x}_k^2(j) + \tilde{y}_k^2(j) + \tilde{z}_k^2(j)]/N)^{1/2} / \\ (\sum_{k=1}^{N}[(x_k(j) - z_{x,k}(j))^2 + (y_k(j) - z_{y,k}(j))^2 + \\ (z_k(j) - z_{z,k}(j))^2]/N)^{1/2} \quad (28)$$

$$\overline{NPE} = \frac{1}{M}\sum_{j=1}^{M} NPE(j) \quad (29)$$

where $j = 1, 2, \cdots, M$ is the Monte Carlo runs, $M = 100$; $k = 1, 2, \cdots, N$ is the simulation steps, $N = 2000$; $\tilde{x}_k(j)$, $\tilde{y}_k(j)$, and $\tilde{z}_k(j)$ are respectively the estimation position error of k time and the j th simulation in Cartesian CS; $x_k(j)$, $y_k(j)$, and $z_k(j)$ are respectively the true position; $z_{x,k}(j)$, $z_{y,k}(j)$, and $z_{z,k}(j)$ are respectively the measurements of three directions.

Four different trajectories (Fig. 1) of the target are tracked in the simulation.

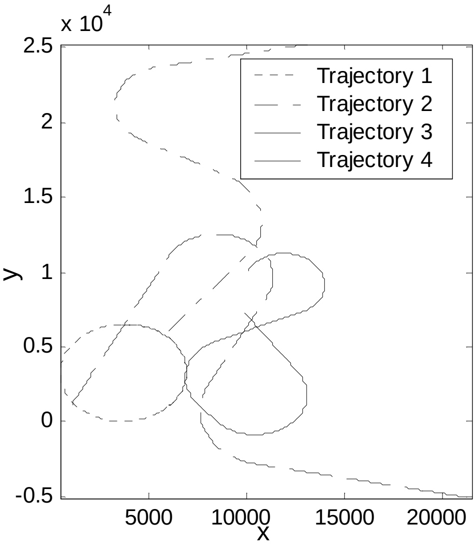

Fig.1 The target trajectories (m)

Trajectory 1: $\boldsymbol{x}_0 = [6000 \quad 200 \quad 1000 \quad 200]^{\mathrm{T}}$, constantly turns with $\omega(t) = 5^{\circ}/s$.

Trajectory 2: $\boldsymbol{x}_0 = [6000 \quad 200 \quad 6000 \quad 250]^{\mathrm{T}}$, 20~40s constantly turns with $\omega(t) = 5^{\circ}/s$ (turn left), 55~75s constantly turns with $\omega(t) = -7^{\circ}/s$ (turn right), other time moves in straight line with constant velocity (uniform motion).

Trajectory 3: $\boldsymbol{x}_0 = [1000 \quad 200 \quad 1000 \quad 400]^{\mathrm{T}}$, 25~45s constantly turns with $\omega(t) = -9^{\circ}/s$ (turn right), 60~75s constantly turns with $\omega(t) = 7^{\circ}/s$ (turn left), other time moves in straight line with constant velocity.

Trajectory 4: $\boldsymbol{x}_0 = [10000 \quad 200 \quad 10000 \quad 250]^{\mathrm{T}}$, 0~25s constantly turns with $\omega(t) = -9^{\circ}/s$ (turn right), 40~55s and 60~85s constantly turns with $\omega(t) = 7^{\circ}/s$ (turn left), other time moves in straight line with constant velocity. This is a trajectory like the figure "8".

6.2 Simulation Results

Simulation results are given by Table 2 to Table 5. Table 2 and Table 3 gave the RMS of x and y directions respectively with ASM, IMM and MAA. Table 4 gave the NEP and Table 5 gave the CL.

Table 2 Root Mean Square of x Coordinate (m)

$\overline{RMS(x)}$	EBDA	IMM	MAA
Trajectory 1	29.7742	26.1995	45.7578
Trajectory 2	28.6938	27.3768	46.8494
Trajectory 3	29.4929	29.9152	44.0302
Trajectory 4	27.8469	34.4717	38.2877

Table 3 Root Mean Square of y Coordinate (m)

$\overline{RMS(y)}$	EBDA	IMM	MAA
Trajectory 1	28.1050	22.7559	47.8447
Trajectory 2	26.1246	32.4014	36.7867
Trajectory 3	30.0558	29.6224	49.4845
Trajectory 4	29.6671	34.7230	50.2352

Table 4 Normalized Position Error (m)

$\overline{NPE}$	EBDA	MBDA	IMM	MAA
Trajectory 1	0.2694	0.3326	0.2404	0.4587
Trajectory 2	0.2692	0.3064	0.2659	0.4226
Trajectory 3	0.2976	0.3211	0.2981	0.4548
Trajectory 4	0.2844	0.3167	0.3438	0.4461

Table 5 Computational Load (s)

CL	EBDA	MBDA	IMM	MAA
Trajectory 1	4.526	4.878	4.976	1.552
Trajectory 2	4.486	5.346	5.045	1.503
Trajectory 3	4.236	4.948	5.107	1.513
Trajectory 4	4.446	5.003	5.318	1.518

The filtering parameters of ASM and MAA are identical and invariant in this simulation. The maneuverability (in Cartesian CS) is stronger and stronger from trajectory 1 to 4. From Table 2, Table 3 and Table 4, the tracking performance of EBDA and MBDA based of ASM is improved obviously to MAA, although from "Table 5" we can see that the computational load of EBDA and MBDA is larger than that of MAA. Here we have 2000 steps of one Monte Carlo simulation, the time of one cycle of ASM is less than 3ms (which is relative to the computer performance), and to 50ms sampling interval the resource is well enough.

For the tracking performance of EBDA, to trajectory 1, IMM is a little better than EBDA, to trajectory 2 and 3 the performances of the two algorithms are near to each other, whereas to trajectory 4 EBDA is better than IMM. EBDA is also better than MBDA. From Table 2, Table 3 and Table 4 we can conclude that when the maneuverability is weak or the motion mode varies little, IMM is a little better, but when the maneuverability is strong or the motion mode varies much, ASM is better. Furthermore to say relatively, the tracking performance of ASM is more stationary, and that of IMM varies more obviously.

In the ASM algorithm the adaptability of α is not considered. We can see from (8) that if the acceleration a is approximately constant then the coefficient α has less effect. In fact, to trajectory 1, the maneuver time of the relative angular motion should be longer, that is α could be smaller. We take $\alpha = 0.01$ for another simulation. The results show that the performance of ASM ($\overline{NPE} = 0.2614$ by EBDA) is nearer to IMM but not so apparent.

5 Conclusion

This paper studied and proposed a new improved adaptive single model and two algorithms for maneuvering target tracking based on the relative angular motion of aerocrafts and radar, which is also suitable for range tracking. To say relatively, the tracking performance of ASM is more stationary, and that of IMM varies more obviously according to the maneuverability and the design of model-set.

The united velocity-acceleration estimation-based decision algorithm and direct measurement-based decision algorithm based of ASM seem to be valid for MTT. The physical concept of these new model algorithms is clear, and they are simple and valuable for practical engineering, especially for radar tracking. Here we emphasized particularly on steadily tracking and exactly estimating of the state in spherical CS, which about the estimation of the state in Cartesian CS is not the goal of this paper, we will study in another one.

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