Prove that the Collatz conjecture is correct

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Abstract—Assuming that the Collatz conjecture is incorrect, the natural numbers can be divided into two groups, namely, set B and set H. Set B is a natural number that satisfies the Collatz conjecture, and set H does not satisfy the Collatz conjecture. After performing the Collatz operation on the numbers in the H group, it is proved that the Collatz conjecture is correct.

Key words: Collatz algorithm ; B set ; H Set
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1. Preparatory work

Collatz algorithm:

$$f(n)\begin{cases} n/2 & \text{if } n \equiv 0(\text{mod}2) \\ 3n+1 & \text{if } n \equiv 1(\text{mod}2) \end{cases}$$

operations. Collatz conjectures that all natural numbers will eventually enter the cycle of $4 \to 2 \to 1 \to 4$ through the algorithm of Collatz algorithm, Suppose the Collatz guess is incorrect. Then there is:

Definition 1 Let N be a natural number set. The N set is decomposed into B set and H set.

1, The natural number that satisfies the " Collatz conjecture" is called the"Collatz number set", which is called the set B.

$B = \{b_1, b_2, b_3, b_4, \ldots \ldots b_i\}$ $b_1 = 1, b_2 = 2,\ b_3 = 3,\ b_4 = 4 \ldots \ldots b_i \to \infty$

2, The natural number that can not satisfy the " Collatz conjecture" is called the "Set of non Collatz numbers". It is called the set H.

$H = \{h_1, h_2, h_3, h_4, \ldots \ldots h_i\}$

Obviously $B \cup H = N$, $B \cap H = \emptyset$, h_1 is an odd number. Since h_1 is the smallest natural number in H sets, that is if $n < h_1$ has $n \in B$.

2. Proof of proposition

Assuming that the Collatz conjecture is incorrect, that $\forall n$ if $n \in H$, Then there are Collatz algorithm

$$f(n)\begin{cases} n/2 & \text{if } n \equiv 0(\text{mod}2) \\ 3n+1 & \text{if } n \equiv 1(\text{mod}2) \end{cases}$$

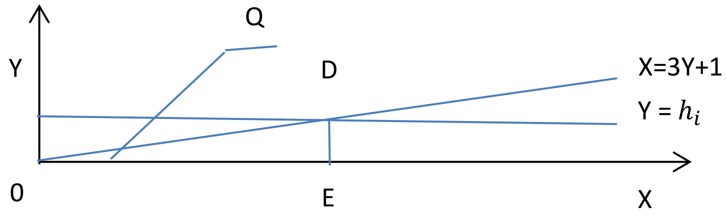
After Collatz algorithm, $f(n) \ldots \ldots \to 4 \to 2 \to 1 \to 4$, will not happen, that $n \in H$.

In $H = \{h_1, h_2, h_3, h_4, \ldots \ldots h_i\}$, h_1 is the smallest number in H, Obviously it is odd number, there is:

Lemma 1 Set $h_1 = 2p + 1$. have $p = 2n + 1$, that is $h_1 = 4n + 3$, and $3h_1 + 1$ cannot be divisible by 4 .

Proof When $p = 2n$, $h_1 = 4n + 1$, Collatz algorithm: $3 \times (4n + 1) + 1 = 12n + 4$, $(12n + 4)/2 = 6n + 2$, $(6n + 2)/2 = 3n + 1$, $\because 3n + 1 < 4n + 1 = h_1$, $\therefore 3n + 1 \in B$, $\therefore p = 2n + 1$ $\because$ $3h_1 + 1 = 3(4n + 3) + 1 = 12n + 10$, and $(12n + 10)/4 = 3n + 5/2$, not an integer, $\therefore 3h_1 + 1$ cannot be divisible by 4. **#**

To facilitate the proof, we establish the plane rectangular coordinate system as follows



In the picture, $Y = h_i$ and $X = 3Y + 1$ intersect at D point. The DE vertical line can be established to get the triangle D0E, have $tan\,Q = \frac{Y}{3Y+1}$, set $DE = Y = h_1$ that is $0E = X_1 = 3h_1 + 1$, (X_i and h_i are one-to-one correspondence) when $DE = h_1$, have $tan\,Q = \frac{h_1}{3h_1+1}$, $\because 3h_1 + 1$ is even number, Collatz algorithm, there are,

$h_2 = \frac{X_1}{2^{n_1}} = \frac{3h_1+1}{2^{n_1}}$, $n_1 \geq 1,\ h_2 \in H$, (When $\frac{3h_1+1}{2^{n_1}}$ is an odd, n_1 is the maximum value), set $y = \frac{3h_1+1}{2^{n_1}} = h_2$, $\because tan\,Q = \frac{h_1}{3h_1+1} = \frac{h_2}{X_2}$ that is: $X_2 = \frac{h_2(3h_1+1)}{h_1}$

$\therefore X_2 = \frac{(3h_1+1)^2}{2^{n_1}h_1}$, Collatz algorithm, there are: $h_3 = \frac{X_2}{2^{n_2}} = \frac{(3h_1+1)^2}{h_1 2^{n_1} 2^{n_2}}$, here $Y = h_3$,

$tan\,Q = \frac{h_1}{3h_1+1} = \frac{h_3}{X_3}$ that is: $X_3 = \frac{h_3\,(3h_1+1)}{h_1} = \frac{(3h_1+1)\,(3h_1+1)^2}{h_1^2 2^{n_1} 2^{n_2}}$, Collatz algorithm, there are: $h_4 = \frac{X_3}{2^{n_3}} = \frac{(3h_1+1)^3}{h_1^2 2^{n_1} 2^{n_2} 2^{n_3}}$ … … Reasoning in the same way , We can get: $h_{i+1} = \frac{(3\,h_1+1)^i}{h_1^{i-1}\,2^{n_1+n_2 \ldots + n_i}}$, $(n_1 + n_2 + \cdots n_i) \geq i,$ … … (1)

When $n_1 = n_2 = \cdots n_i = 1$, $(n_1 + n_2 + \cdots + n_i) = i$, set $(n_1 + n_2 + \cdots n_i) = \beta,\ \beta \geq i$

That is (1) have: $h_{i+1} = h_1\left(\frac{3h_1+1}{h_1 2^{\beta/i}}\right)^i$ $\because h_{i+1}$ is an integer, $\therefore \frac{3h_1+1}{h_1 2^{\beta/i}}$ must is an integer, $\because \frac{3\,h_1+1}{h_1}$ not an integer, $\therefore \frac{3h_1+1}{2^{\beta/i}}$ must is an integer, Lemma 1: 4 do not divide $(3\,h_1 + 1), \therefore \exists\, 2^{\beta/i} = 2,\ \therefore\ \beta/i = 1,\ \beta = i,\ \therefore\ (n_1 + n_2 \ldots + n_i) = i,\ \therefore n_1 = n_2 = \cdots n_i = 1,$
$\therefore h_i = 4n + 3,\ \therefore\ p_i = 2n_j + 1,\ \therefore h_1 = 2p_1 + 1 = 2(2(2 \ldots (2n_j + 1) \ldots + 1) + 1) + 1,$
$j = 1,2,3 \ldots\ j \to \infty$

proof: If j is not infinite, then loops must occur in H sets, and (1) the necessary conditions for loops to occur are: $h_1\left(\frac{3h_1+1}{h_1 2^{\beta/i}}\right)^i = h_1$, that is $\left(\frac{3h_1+1}{h_1 2^{\beta/i}}\right)^i = 1$, $\therefore 3h_1 + 1 = h_1 2^{\beta/i}$

Obviously, $3h_1 + 1 \neq 2h_1$, Therefore, equation (1) cannot generate cycles beginning with h_1 . $\because n_1 = n_2 = \cdots n_i = 1$, so $\forall h_i$ will be the same result, $\therefore j \to \infty$, **#**

Because $j \to \infty, \therefore h_1 = 2p_1 + 1 = 2\big(2\big(2 \ldots \big(2n_j + 1\big) \ldots + 1\big) + 1\big) + 1 \to \infty$, So in H set , we can never find a h_1 to satisfy Collatz operation, that is $H = \emptyset$, Therefore, the Collatz conjecture is correct. **Proof of completion!**

REFERENCES

[1] Chaohao Gu, Mathematics Dictionary, *Shanghai Dictionary Press*, (1992)