2022 MCM
Problem A: Power Profile of a Cyclist

https://commons.wikimedia.org/wiki/File:Dave_Zabriskie_-_USA_Pro_Time_Trial.jpg

Background

There are many types of bicycle road races including a **criterium**, a team time trial, and an **individual time trial**. A rider's chance of success can vary for these contests depending on the type of event, the course, and the rider's abilities. In an individual time trial, each individual cyclist is expected to ride a fixed course alone, and the winner is the rider who does so in the least amount of time.

An individual rider can produce different levels of power for different lengths of time, and the amount of power and how long a given amount of power a rider can produce varies greatly between riders. A rider's **power curve** indicates how long a rider can produce a given amount of power. In other words, for a particular length of time the power curve provides the maximum power a rider can maintain for that given time. Generally, the more power a rider produces, the less time the rider can maintain that power before having to reduce the amount of power and recover. A rider may choose to briefly exceed the limits on their power curve, but the rider then requires extra time at a lower power level to recover. Moreover, a rider's power output in the past matters, and riders are increasingly fatigued as a race progresses.

Riders are always looking to minimize the time required to cover a given distance. Given a particular rider's capability according to that rider's power curve, how should that rider apply power while traversing a given time trial course? Additionally, many types of riders may participate in an individual time trial, such as a **time trial specialist**, a **climber**, a **sprinter,** a **rouleur**, or a **puncheur**, and each type of rider has a distinct power curve.

Requirement

Develop a model that can be applied to any type of rider that determines the relationship between the rider's position on the course and the power the rider applies. Keep in mind that the rider has a limit on the total energy that can be expended over the course, as well as limits that accumulate from past aggressiveness and for exceeding the power curve limits.

Your model development and report should include the following:

- Define the power profiles of two types of riders. One of your riders should be a time trial specialist, and the other is a rider of a different type. You should also consider profiles of riders of different genders.
- Apply your model to various time trial courses including, at a minimum, the ones listed below for each power profile you defined above:
    - 2021 Olympic Time Trial course in Tokyo, Japan,
    - 2021 UCI World Championship time trial course in Flanders, Belgium,
    - At least one course of your own design that includes at least four sharp turns and at least one nontrivial road grade. The end of the course should be near its start point.
- Determine the potential impact of weather conditions, including wind directions and wind strengths, to determine how sensitive your results are for small differences in the weather and environment.
- Determine how sensitive the results are to rider deviations from the target power distribution. It is unlikely that a rider can follow a highly detailed plan and will miss the power targets. The rider and the **Directeur Sportif** will have some idea of the possible range of expected split times at key parts of a given course.
- Discuss how to extend your model to include the optimal power use for a team time trial of six riders per team, where the team's time is determined when the fourth rider crosses the finish line.

As part of your solution, write a two-page rider's race guidance for a Directeur Sportif of a team. The rider's race guidance should focus on the results for one rider and one time trial course. It should contain an overview of the directions for the rider. It should also include a broad summary of your model but be appropriate for a Directeur and rider who do not have backgrounds in mathematics.

Your PDF solution of no more than 25 total pages should include:

- One-page Summary Sheet.
- Table of Contents.
- Your complete solution.
- Two-page rider's race guidance for a Directeur Sportif.
- Reference List.

Note: The MCM Contest has a 25-page limit. All aspects of your submission count toward the 25-page limit (Summary Sheet, Table of Contents, Reference List, and any Appendices). You must cite the sources for your ideas, images, and any other materials used in your report.

Glossary

Criterium: a bicycle race that takes place on a closed course. The length can be specified by a fixed number of laps or the most laps in a predetermined time period.

Directeur Sportif: a team's director who is responsible for managing the riders and staff, making race decisions, and deciding the team composition for a given race.

Individual Time Trial: an event in which riders traverse a predetermined course one at a time. The riders are not allowed to work together or ride near one another. The time required to traverse the course is recorded for each rider. The lower the time the better the rider's final placement.

Power Curve: is a visual representation of the maximum power a rider can maintain for a particular length of time.

Glossary of Rider Types

Climber: a rider that specializes in races that have multiple long climbs.

Puncheur: a rider that specializes in races that include many short, steep climbs or many sharp accelerations.

Rouleur: a rider that is a generalist and can do well in races with a wide variety of terrains.

Sprinter: a rider that specializes in producing extremely high power for short periods of time. These riders generally focus on winning at the end of a race or during the intermediate sprints (if a race has intermediate sprints).

Time Trial Specialist: a rider that specializes in the individual time trial events.